AN EFFICIENT FREQUENCY DOMAIN EQUALIZATION METHOD FOR CHANNELIZER-BASED SOFTWARE DEFINED RADIOS

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Abstract - In this paper we present an efficient method for equalizing the wireless channel in DSP-based radio architectures. Equalization is usually performed in the time domain via transversal filters. Our proposal performs the channel equalization in the frequency domain by adjusting scalar gains and phases of each path of a perfect reconstruction filter bank. In this process the spectral products form a composite channel-equalizer and matched filter frequency response with uniform gain and phase profiles. This process cancels severe signal distortion caused by the multipath channel. The appropriate gain and phase profiles can be obtained adaptively or from a channel probe. In this paper the frequency domain equalization method is applied in fully digital receivers containing perfect reconstruction (PR) polyphase filter banks designed to synthesize arbitrary filter responses. The output ports of an M-path analysis channelizer present samples of the narrow band complex envelope from spectral segments spanning the entire signal bandwidth. Appropriate scalar gains and phase shifts are applied to each path to reverse the distortion caused by the wireless channel. An estimation block guides the selection of the proper parameter values. Trade-off considerations between total workload and receiver performance influence the selection of the number of IFFT spectral sample points in the analysis channelizer. The channelizer spectral response interacts with the applied gains to form a “connect the dots” piece-wise linear approximation to any specified frequency response. Theoretical analysis and simulation results, which prove the efficacy of the proposed method, are provided in this paper along with suggestions for further research and development work.

Keywords – Frequency domain equalization, cascade analysis and synthesis polyphase channelizers, perfect reconstruction filter bank, software defined radio, digital receiver, rectangle and triangle shaped spectral channels

1. INTRODUCTION

The design of software defined radio receiver can include an analysis polyphase down converter channelizer and multiple synthesis up converter channelizers [1]-[5]. The primary advantage of such an architecture is its ability, in a single structure, to simultaneously down convert multiple received spectra, with arbitrary bandwidths, frequency locations and modulation formats. The process avoids replication of the digital down conversion section and offers significant improvement in computational efficiency of the receiver. The appropriate design of the first tier analysis channelizer can be selected from a wide set of variations. The most standard down converter channelizer architecture is composed of an M-path partitioned filter which is followed by M-point IFFT block. An input commutator is responsible for delivering the input data to the engine in the appropriate order. In a common option an M-path down converter channelizer performs M-to-1 down sampling of the input time series while simultaneously aliasing all the M channels to base-band. The one option we favor, as being the most suitable one for software radio applications, performs M/2-to-1 down sampling of the input time series while shifting, primarily by aliasing, all the M channels to base-band. The doubled output sample rate formed by this version of the down converter channelizer avoids the spectral folding problem which occurs when the output sample rate is maximally decimated to the channel spacing.
The second tier \( P_n \)-path polyphase up converter channelizers which are used in the signal bandwidth synthesis process, recompose at baseband, when necessary, the fragments, belonging to signals whose bandwidths are wider than the analysis channelizer channel bandwidths. The version of the up converter channelizer selected, among the many available, for the receiver design that we consider in this paper is the dual form of the M/2-to-1 analysis down converter channelizer. This engine performs 1-to-\( P_n/2 \) up sampling of the input time series. SQR Nyquist filters can be used as low-pass prototype filter in both the analysis and synthesis channelizers, to form a composite Nyquist pulse response which achieves perfect reconstruction (PR). A more flexible option uses a Nyquist filter in the analysis channelizer and a wider bandwidth low pass filter in the synthesis channelizer. The PR property of the Nyquist filters plays a fundamental role in the synthesis of the arbitrary filter and in particular the equalization process. A selector, placed between the analysis and synthesis blocks, commutes the output ports of the first tier analysis channelizer that contains spectral fragments of the same signal bandwidths and properly delivers them to the up converter synthesis channelizers that reassemble them at baseband. More details on polyphase up and down converter channelizers can be found in [1], while software radio receiver designs based on polyphase channelizer engines can be found in [4]-[5]. Since the focus of this paper is the equalization process, for simplicity, in the following, we will consider a single signal bandwidth as input to the system. This selection does not impact design issues and/or performance results.

When the received spectrum has been shifted to baseband and its sample rate has been properly reduced the channel equalization must be performed before recovering the transmitted symbols. The challenging task is to embed the equalization process in channelizer-based software defined radios minimizing the total workload and maximizing their performance. Fighting the artifacts caused by the multipath channel is a primary goal of mobile wireless transmitters and receivers, and performance and efficiency are the main aspects to consider in the design process. In this paper we present an efficient frequency domain equalizer that can be embedded in PR channelizer-based software defined radio architectures.

Frequency domain equalizers are not new in the literature. It is well known [6], [7] that the effect of the multipath channel can be modeled as a distorting filtering process which affects both gain and phase profile of the transmitted signal spectrum. Orthogonal frequency division multiplexing (OFDM) modulation is the best example of successful frequency domain equalization. Since the FFT bins of an OFDM system are orthogonal, and by virtue of the cyclic prefix, these signal components have been circularly convolved with the channel, the channel equalization can be performed as independent scalar gain and phase adjustments to spectral component. In this paper we apply a related idea to channelizer-based receivers even when the decomposed channels are not orthogonal nor have the signal components been circularly convolved with the channel.

At the output port of the first tier down converter channelizer, prior to synthesis, a set of complex scalars with appropriate gains and phase shifts is applied to each channelizer channel in order to compensate for the distorting effect of the multipath channel to form a composite channel-equalizer frequency response which has uniform gain and phase. This version of the system requires knowledge of the channel obtained by a channel probe. A channel control block adjusts the set of gain and phase rotators according to the knowledge of the wireless channel. Other forms of the equalization process can be implemented by adaptively updating estimates of the channel response.

In this paper we will not consider the noise coloring effect of the proposed equalizer on the additive white Gaussian noise which enters the system along with the signal. When the white noise spectrum is multiplied with the set of coefficients it becomes colored and further filtering tasks can be performed in the system in order to whiten it.

The performance of channelizers with two spectral channel shapes, hence with two impulse response shapes, are presented and analyzed in this paper. Both options produce a perfect reconstruction Nyquist pulse. Unfortunately, designs based on the standard rectangle channel shape, the one commonly used for channelizer prototype filters, does not result in a good linear approximation when different gains are applied to adjacent channels. The triangle channel shape is
derived and used as the preferred prototype filter for the proposed design.

This paper is organized in six main sections. Section I introduces the equalization problem for wireless communication systems and provides an overview of the proposed solution. Section II provides the theoretical background which is needed to fully appreciate the proposed design. First the basics on wireless signal model and equalization process are given and then the perfect reconstruction (PR) property of the analysis-synthesis chain is demonstrated. Section III presents the frequency domain equalization from a filtering point of view while the proposed design is shown and explained in Section IV. Section V provides the simulation results that demonstrate its correct functionality. The conclusions along with suggestions for further developments are given in Section VI.

II. BACKGROUND

It is well known that a multipath channel causes inter-symbol interference (ISI) to the received signal. In receivers operating on quadrature amplitude modulated (QAM) signals, the most widely used technique for mitigating the undesired channel effects is to use a finite impulse response (FIR) equalizer following the matched filter as shown in Figure 1. The FIR equalizer can be designed subject to different criteria, among which, the zero forcing and minimum mean squared error (MMSE) are the most commonly used design criteria. The final goal is to have the overall cascade response of the shaping filter, channel, matched filter and the equalizer form a Nyquist channel.

\[ \hat{r}(n) = \sum_{m=0}^{N-1} h_l(m) r(n - m) \]  

(2)

Denoting the transmitted signal to be \( s(n) \), the received signal \( r(n) \) can be written as:

\[ r(n) = s(n) * c(n) + w(n) \]  

(1)

where, \( c(n) \) is the tapped delay line channel coefficients and \( w(n) \) is the additive white noise. In the time domain, the channel \( c(n) \) introduces multiple delayed, gain and phase modified versions of the transmitted signal which causes the ISI. When viewed in the frequency domain, the channel \( c(n) \) modifies the gain and phase of the transmitted signal’s spectrum. This fact suggests that the equalization can be done in either time or frequency domain. Because of the implementation simplicity, the time domain linear equalizer, i.e., FIR filter, method is frequently used in many applications. In this paper, we follow a different approach and propose an efficient frequency domain equalizer structure based on polyphase perfect reconstruction channelizer described in [3]-[5].

Figure 2 shows the theoretical model of the proposed equalizer which is a cascade of an analysis and a synthesis filter bank. The M-path analysis channelizer decomposes the input signal spectrum into M equally spaced, equal bandwidth sub-channels while performing \( \frac{M}{2}:1 \) down-sampling of the input time series. The synthesis channelizer reassembles the M sub-channels while performing \( 1: \frac{M}{2} \) up-sampling. Let \( h(n) \) be the analysis channelizer prototype low-pass filter of length \( N_2 \); and \( g(n) \) be the synthesizer channelizer prototype low-pass filter of length \( N_2 \) where \( N_1 \) and \( N_2 \) are usually designed to be odd numbers. Then, the analysis filter \( h_l(n) \) for the \( l^{th} \) path can be written as \( h_l(n) = h(n)e^{j\omega_l n} \), where \( \omega_l = \frac{2\pi n}{M} \) for \( l = 0,1,\ldots,M-1 \). Similarly, the \( l^{th} \) path synthesis filter can be written as \( g_l(n) = g(n)e^{j\omega_l n} \). The signal at the output of the \( l^{th} \) analysis filter, denoted as \( r_l(n) \), can be written as:

\[ r_l(n) = \sum_{m=0}^{N_l-1} h_l(m) r(n - m) \]  

\[ r_l(n) = r_l(\frac{M}{2} n) e^{-j\omega_l \frac{M}{2} n} = r_l(\frac{M}{2} n) e^{-j\pi m} = \begin{cases} r_l(\frac{M}{2} n), & l \text{ is even} \\ r_l(\frac{M}{2} n) \cos(n\pi), & l \text{ is odd} \end{cases} \]  

(3)
We see from Eq. (3) that after the \( \frac{M}{2} \cdot 1 \) down sampling, all the even indexed channels alias to the baseband and all the odd indexed channels alias to the half sample rate. Note that, the odd indexed channels can be translated to the baseband trivially by multiplying with \( \cos(n\pi) \) after the down sampling process. The phase shift can also be applied as a circular shift of buffers between the filters and the IFFT based phase rotators in a polyphase filter version of the filter bank.

Ignoring for now the intermediate equalization and matched filtering processing steps that will be taken into account in the next section of this paper, \( \hat{y}_l(n) \) is initially \( \frac{M}{2} \) zero packed and then fed into the \( l^{th} \) path synthesis filter. The \( \frac{M}{2} \) zero packing creates \( \frac{M}{2} \) spectral copies and the synthesis filter \( g_l(n) \) selects the one centered on \( \omega_l \) and rejects all other copies. The \( \frac{M}{2} \) sampling rate reduction allows the intermediate signal processing blocks to operate at reduced sample rate which makes the proposed structure computationally efficient.

By following the same reasoning, the overall PR channelizer output (analysis-synthesis chain) signal \( y(n) \) can be expressed as:

\[
y(n) = \sum_{l=1}^{M-1} r(n) * h_l(n) * g_l(n) = 
\]

\[
= \sum_{l=1}^{M-1} \left\{ \sum_{m=0}^{N_l-1} h(m) e^{j\omega_l m} g(n-m)e^{j\omega_l (n-m)} \right\} * r(n) 
\]

\[
= \sum_{l=1}^{M-1} \left[ (h(n) * g(n))e^{j\omega_l n} \right] * r(n) 
\]

\[
= \sum_{l=1}^{M-1} h_{NYQ}(n)e^{j\omega_l n} * r(n) \tag{4}
\]

where \( h_{NYQ}(n) = h(n) * g(n) \), is the Nyquist pulse. Equation (4) explicitly shows that the PR channelizer implements a bank of \( M \) equally spaced Nyquist filters each centered on multiples of \( \omega_l \). The frequency response of the adjacent Nyquist filters cross at their half amplitude points (-6 dB in power), and exhibit odd symmetry with respect to the crossover point. Thus, we obtain the perfect reconstruction property, shown in Eq. (5).

\[
y'(n) = \sum_{l=1}^{M-1} h_{NYQ}(n)e^{j\omega_l n} * r(n) = r(n) * \delta(n - \frac{N_1 + N_2}{2}) \tag{5}
\]
III. FREQUENCY DOMAIN FILTERING

In this section, we add the intermediate processing block between the analysis and synthesis channelizer. The intermediate processing section is composed of complex scalar multiplier arrays. Each multiplier array modifies the gain and phase of each analysis channelizer output, which is equivalent to modifying the spectrum of the input signal in the piece-wise linear sense, thus performing frequency domain filtering. Because the system is linear time invariant, the filtering operations in the sense of the multiplier arrays can be cascaded in any order. In the selected case we are examining, the receiver needs a matched filter and an equalizer filter, thus two multiplier arrays are needed to accomplish the task. By including the intermediate processing section, the PR channelizer output $y(n)$ can be expressed as:

$$y(n) = \sum_{l=1}^{M-1} c_l H_{MF}(l) h_{NYQ}(n) e^{j \omega_l n} * r(n)$$

where, $c_l = \sum_{l=0}^{M-1} c(n) e^{-j \omega_l n}$, is the $M$ point DFT of discrete channel taps; and $H_{MF}(l) = \sum_{l=0}^{M-1} h_{MF}(n) e^{-j \omega_l n}$ is the $M$ point DFT of the receiver’s matched filter impulse response $h_{MF}$. The output signal $y(n)$ is now the equalized and matched filtered output signal.

To demonstrate the frequency domain filtering operation, let us set $C_l = 0, l = 0...M - 1$ for this moment and only examine the matched filter response. Although, any Nyquist channel $h_{NYQ}(n)$ provides us perfect reconstruction in terms of analysis and synthesis channelization process, the performance of the frequency domain filtering may vary greatly. This difference is due to the fact of using Nyquist pulse based piece-wise linear spectrum approximation. In the following paragraphs we demonstrate the frequency domain filtering processing based on two types of Nyquist pulses: the rectangular shaped spectrum approximation and the triangular shaped spectrum approximation.

Figure 3 shows the comparison between the channelizer synthesized frequency domain matched filter (in blue) and its prototype matched filter (in red) based on the rectangular shaped filter design. The upper subplot of Figure 3 shows the time aligned and overlaid impulse response of the two filters. It can be seen that the channelizer synthesized filter has much longer delay than the prototype filter. We also observed that, within the two filters’ overlap region, their impulse responses achieve excellent matches. The lower subplot of Figure 3 compares the frequency responses of the two filters. We can see the channelizer synthesized filter is the piece-wise linear approximation by the modulated channelizer’s prototype Nyquist pulses whose gain and phase are modified by the multiplier array. Moreover, this spectrum approximation is valid as long as the number of channelizer paths $M$ is longer than the prototype filter length, i.e., $M=64$ is greater than the matched filter length in this case.

Figure 3. Rectangular Approximation Based Channelizer Synthesized Matched Filter vs. Prototype Matched Filter.

Figure 4. Triangular Approximation Based Channelizer Synthesized Matched Filter vs. Prototype Matched Filter.

Figure 4 shows the similar plot as Figure 3 but with triangular shape spectrum based design. As we can see from the lower subplot of Figure 4,
the triangular based design offers much smoother spectrum approximation than the rectangular based design. The improvement is due to the fact that when different gains are applied to adjacent channels, the triangular based design performs linear straight line interpolation between these points. Based on the previous discussion, we now conclude that, under proper arrangement, by incorporating the multiplier arrays in between the analysis and synthesis channelizers, we are able to perform frequency domain filtering with PR channelizer.

IV. PROPOSED ARCHITECTURE

Figure 5 shows the design of the proposed equalizer architecture which is formed by a PR analysis-synthesis chain to which two sets of multiplier arrays has been added. As anticipated, the analysis filter bank disassembles the input spectrum while simultaneously shifting its fragments to base-band. The dimensionality of such an engine is designed according to the input signal. Also, workload issues help in the proper selection of the number of paths. Once the signal fragments are at base-band and their sample rate has been properly decreased (it is now commensurate to the new reduced bandwidth) we can equalize them via multiplier arrays. The equalization process is performed in the frequency domain by applying appropriate complex scalars to the output ports of the analysis channelizer. The complex scalars adjust the gain and phase of each channelizer channel undoing the effect of the wireless channel. The complex multipliers are selected according to the prior knowledge of the multipath channel which is provided to the system by a channel probe block.

Before synthesizing the equalized spectral fragments we perform matched-filtering on the signal in the frequency domain by applying its spectral response to its set of multipliers. At the output of the synthesis channelizer, additional post processing may be applied to the filtered signal which might include arbitrarily interpolation to obtain two samples per symbol [4].

V. SIMULATION RESULTS

We describe our simulation results in this section. Figure 6 shows the impulse response, and the analysis filter spectra of the designed channelizer based on the triangular shaped spectral design. As expected the channelizer’s overall impulse response is a Nyquist pulse. We emphasize the importance of requiring the analysis filter spectra to be a triangle. Figure 7 shows the transmitted QPSK signal constellation and its power spectrum. In the absence of channel and noise, the received and demodulated QPSK signal via the designed channelizer is shown in Figure 8, where we can see the channelizer successfully accomplished perfect reconstruction as well as frequency domain matched filter tasks.

In Figure 9 we apply a channel whose distortion profile is also displayed in the same figure. We can see, from the distortion profile, the channel frequency variable gain has modified the power spectrum of the received signal and that this modification will be the cause of ISI. Figure 10 demonstrates the distorted signal constellation as well as finer resolution detail of the distorted power spectrum.
Assuming the receiver has perfect knowledge about the channel, we then apply the equalization as the frequency domain inverse of the probed channel via the equalizer multiplier array. The equalized results are explicitly shown in Figure 11. We can see the channel introduced constellation dispersion has been significantly shrunk. The lower subplot of Figure 11 shows spectra of the channel (pink line), the equalization multiplier array response (black line), and the equalized and matched filtered signal spectra. We can see from the spectra, the channel distortion has been dramatically mitigated.

Figure 7. Transmitted Signal Constellation and Spectrum.

Figure 8. Demodulated Signal Without Channel and Noise.

Figure 9. Channel Distortion Profile.

Figure 10. Received Constellation and Power Spectrum.

Figure 11. Equalized and Matched Filtered Results.
VI. CONCLUSIONS

In this paper we have presented a frequency domain equalization method that can be applied to PR channelizer-based software defined radios. It is well known that polyphase filter bank structures are suitable candidates for fully digital transmitting and receiving architectures. The fundamental issue of embedding the wireless channel equalization as well as the matched filter in such architectures has been addressed in this paper and a frequency domain equalization method based on the perfect reconstruction property of the analysis and synthesis chain has been proposed. The equalization is performed by applying appropriate gains and phase shifts at the output ports of the first tier analysis channelizer in order to undo the effect of the channel. The method proposed here is based on knowledge of the wireless channel which is provided to the system by a channel probe signal. Further research developments are directed towards the inclusion of additional stages in the proposed architecture for adaptive, preamble free, techniques to acquire the channel.

REFERENCES


