A SUBCARRIER AND BIT ALLOCATION ALGORITHM FOR MOBILE OFDMA SYSTEMS

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Abstract – OFDMA technique is becoming a popular solution to increase the spectral efficiency of multi-user wireless digital communication standards, such as WiMAX, LTE and IEEE 802.22. The flexibility of this technique, allied with its robustness against frequency selective time variant channels, makes it a suitable solution for high data rate mobile communication systems. However, there are several challenges that must be overcome in order to achieve high spectrum efficiency. One of these challenges is the bit and subcarrier allocation for each user. There are a lots of algorithms described in literature, ones that only consider the channels conditions, which means that the QoS of each user does not play a rule in the resource allocation process. Such algorithm may result in an unfair distribution of the resources between the users. Others algorithms take into account both the QoS and the channel conditions, but that employ several steps algorithms that may increase the complexity. The aim of this paper is to present a simple subcarrier and bit allocation algorithm that uses the individual channel conditions and QoS to distribute the system resources between the users. If the number of necessary subcarriers is smaller than the total number of available subcarriers, than it is possible to use three different priority approaches to distribute the extra subcarriers between the users. The spectral efficiency of each priority approach will be analyzed by using computational simulation.

Keywords – Bit Allocation, Power Allocation, OFDM.

1. INTRODUCTION

Today, personal mobile data communication systems require high spectrum efficiency in a frequency selective time variant channel. Orthogonal Frequency Division Multiplexing (OFDM) \([1]\) is being used to combat the effects of a multipath channel, but in a multiuser environment, it is inefficient to allocate an entire OFDM symbol to a single user because usually a single user does not require all subcarriers to transmit its own information. One solution to increase the data transmission efficiency is to share the subcarriers with multiple users. This technique is called Orthogonal Frequency Division Multiple Access (OFDMA) \([2]\) and it is being used in several modern mobile communication systems, such as Long Term Evolution (LTE) and Worldwide Interoperability for Microwave Access (WiMAX) \([3]\).

In order to obtain the best results, the conditions of the propagation channel of each user must be considered during the subcarrier allocation process, which means that the Channel State Information (CSI) of each user must be known \([4]\).

There are different resource allocation algorithms and adaptive modulation techniques for OFDMA systems available in literature \([4]\) \([5]\) \([6]\) \([7]\).

The authors in \([4]\) analyze the resource allocation problem for the uplink in a mobile communication system that employs OFDMA. Three algorithms are used to maximize the total data rate between users: the first algorithm defines the number of carriers for each user; the second algorithm performs the subcarrier allocation considering the CSI of each user and; the last performs the power allocation using the Water-filling Theorem \([8]\) approach.

In \([5]\), the authors proposes an approach based on changing the number of bits per symbol transmitted in each subcarrier as a function of the average bit error rate (BER) evaluated over all subcarriers. Basically, the algorithm computes the best modulation order for each subcarrier based on the received signal to noise ratio (SNR), aiming to keep the quality of service constant.

The authors in \([6]\) present a computational efficient algorithm to allocate resources for different users in an OFDM system. The main idea in this proposal is to divide the resource allocation process in two steps: first, the number of subcarriers for each user are estimated; second, the subcarriers are distributed to them. The performance of this approach is slightly worse than the classical solution but the complexity and required time to process the resource allocation algorithm is significantly reduced. To distribute the bits for the allocated subcarriers is done by a classical power and bit allocation algorithm for single user. The same authors presented a variant of this approach in \([9]\), where a multi-user power allocation algorithm based on Water-filling Theorem has been used instead the single-user resource allocation algorithm.

As one can see, the major objectives in the resource allocation algorithms presented in literature are to maximize the overall throughput of the system or
minimize the total transmitted power.

The aim of this paper is to present a subcarrier and bit allocation algorithm that considers the Quality of Service (QoS) requested by each user and the CSI to guarantee that every user will achieve the desired data rate with high probability of success. Three different approaches will be considered in this paper. The first approach will distribute the minimum number of subcarriers that guarantee the minimal data rate for each user. The extra subcarriers will be randomly assigned to the users. In the second approach, a specific subcarrier will be allocated to the user that presents the best channel condition at frequency of this specific subcarrier. Finally, the third method will allocate the extra subcarriers to the user that can achieve the highest data rate. All the techniques considered will be compared through computational simulation.

This paper is organized as follow: Section 2 presents the System Model, while Section 3 presents the adaptive allocation algorithm and Section 4 presents the final conclusions and remarks of this paper.

2. SYSTEM MODEL

The technique proposed in this paper is suitable to perform resource allocation in the downlink and uplink as well. Nevertheless, only the uplink scenario will be exploited in this paper. In this case, the Base Station (BS) is responsible for receiving the signals from all users and performs the subcarrier and bit allocation algorithm using the channel estimation and the QoS for each user. Figure 1 presents a simplified block diagram of the BS.

![Figure 1. Block diagram of the BS.](image)

In this model, the signal received in the BS from all users are aligned and applied to FFT block. The User Carrier Selection block separates the received data symbols in different streams, one for each user. The demodulator receives the individual streams and estimates the received data bit of all users. The User Carrier Selection also separates the pilot subcarriers that will be used by the CSI Estimation block to estimate all channel conditions. The CSI will be applied to the Adaptive Subcarrier and Bit Allocation block that also uses the QoS and data rate required by the users, to establish which subcarrier will be allocated for each user, as well as the number of bits per symbol (modulation order). The results obtained from the subcarrier and bit allocation algorithm are transmitted to the users.

Figure 2 presents the block diagram of the user terminal. The bits to be transmitted by the user are converted from serial to parallel format and the resultant streams feed an Adaptive Mapper that generates the symbols that will modulate each subcarrier. The number of bits per symbol of each subcarrier, the number of subcarrier for each user and the position of the subcarriers in the IFFT frame are defined by the Subcarrier and Bit Allocation Information Recovery block, which recovers all these information from the signal sent by the BS.

![Figure 2. Simplified block diagram of the user terminal.](image)

In this paper, it has been considered that the delay between the received signals from the users and the transmission of the subcarrier and bit allocation data to the users is several orders of magnitude smaller than the channel coherence time [10]. Also, it is assumed that all user’s channels to be time variant, frequency selective and statistically independent [11]. The number of subcarriers used in this system is large enough to guarantee that the bandwidth of each subcarrier is smaller than the channel coherence bandwidth [1].

The received signal from the user \( u \) at the \( n \)-th subcarrier can be state as

\[
c^\prime_{u,n} = H_{u,n} c_{u,n} + v_n,
\]

where \( c_{u,n} \) is the data symbol transmitted by the user \( u \) at the \( n \)-th subcarrier, \( H_{u,n} \) is the channel frequency response of the channel between user \( u \) and the BS at the frequency of the \( n \)-th subcarrier and \( v_n \) is the Additive White Gaussian Noise sample with zero mean and variance \( \sigma^2 \). The data symbol, \( c_{u,n} \), belongs to a \( M\)-QAM constellation with \( M = 2^{2h_{u,n}} \) symbols, where \( b_{u,n} \) is the maximum spectral efficiency that user \( u \) can be achieve at the frequency of the \( n \)-th subcarrier. The main problem of the subcarrier and bit allocation algorithm is to define which subcarriers shall be allocated to each user and how to distributed a limited power through all subcarriers in order to better use the frequency selective time variant channel with channel frequency response \( H_{u,n} \). Next section will present some algorithms that can...
be used to solve this problem.

3. Resource Allocation Algorithms

This section compares the power and spectral efficiency of two algorithms, called Water-filling Algorithm (WA) and Two-step Algorithm (TSA), and also presents a new proposal based on WA, that is called Modified Water-filling Algorithm (MWA). The purpose of the MWA is to maximize the system throughput considering the individual user data rate as a constraint for the optimization problem.

3.1. Water-filling Algorithm

For single user, the power necessary for the \( n \)th subcarrier to transmit \( b_n \) bits per symbol is given by [12]

\[
P_n = \frac{\Gamma \times \sigma_n^2}{(H_n)^2} \times (2^{b_n} - 1)
\]

where

\[
\Gamma = \frac{1}{3} \times \left( Q^{-1} \left( \frac{P_{es}}{4 \times (1 - \frac{1}{\sqrt{N}})} \right) \right)^2
\]

is the SNR Gap [9]. \( P_{es} \) is the target symbol error probability and \( Q^{-1}(\cdot) \) is the inverse \( Q \) function [13].

If the available power for the \( n \)th subcarrier is limited, then the number of bits per symbol that can be transmitted in this subcarrier can evaluated from (2). Thus, if the total available power for all subcarriers is a constant given by

\[
P = \sum_{n=1}^{N} P_n,
\]

where \( P_n \) is the available power for the \( n \)th subcarrier, then the task of defining how to distribute the available power to all subcarriers aiming to maximize the throughput becomes an optimization problem. If the data rate per subcarrier is not a constraint, one optimal solution for this problem is given by [8]

\[
P_n = K - \frac{\Gamma \times \sigma_n^2}{(H_n)^2},
\]

where

\[
P + \sum_{n=1}^{N} \frac{\Gamma \times \sigma_n^2}{(H_n)^2} = K,
\]

is a constant called Water Level and \( N \) is the number of subcarriers. The solution presented in (5) is also known as Water-filling Algorithm.

In the WA [9], the power is allocated to the users aiming to maximize the system throughput. It means that the user with better channel conditions will receive a higher number of subcarriers, while the user with the worse channel conditions will receive a lower number of subcarriers or, eventually, it will not receive any subcarrier. This outcome is possible since the WA does not consider the user’s QoS as an initial constraint. Figure 3 presents the subcarriers and bit distribution for three users using WA.

![Figure 3. Subcarrier and bit allocation using Water-filling Algorithm](image)

The system parameters used in this simulation are presented in Table 1.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total number of subcarriers</td>
<td>256</td>
</tr>
<tr>
<td>Available power over noise variance</td>
<td>37 dB</td>
</tr>
<tr>
<td>Impulse response – user 1</td>
<td>( h_1 = 0.98[n] + 0.45[n - 4] - 0.28[n - 7] + 0.16[n - 9] )</td>
</tr>
<tr>
<td>Impulse response – user 2</td>
<td>( h_2 = 0.88[n] + 0.50[n - 3] + 0.28[n - 7] - 0.15[n - 9] )</td>
</tr>
<tr>
<td>Impulse response – user 3</td>
<td>( h_3 = 0.96[n] + 0.60[n - 2] - 0.48[n - 5] + 0.10[n - 9] )</td>
</tr>
<tr>
<td>Minimal data rate per user</td>
<td>30 bits per symbol</td>
</tr>
</tbody>
</table>

Table 1. Simulation parameters.

It is possible to notice that the number of subcarriers allocated to user 1 and 2 are very low because the channel conditions that these users are experimenting are severe.

3.2. Two Step Algorithm

The Two Step Algorithm [6] considers the channel conditions, the minimal data rate (or QoS) of each user and the available modulation orders. This algorithm operates in two steps. First, based on minimal requested data rate, the algorithm calculates how many subcarriers must be assigned to each user, considering the most robust constellation. If there are more subcarriers
available, they are distributed between the users. Once all subcarriers have been distributed, the second step takes places. In this step, the subcarriers are allocated in the OFDM symbol, considering the channel frequency response of each individual user. Figure 4 shows the subcarriers and bit distribution between three users with equal QoS using the TSA. The parameters used in this simulation are also presented in Table 1. It is possible to conclude that TSA results in a fair distributed subcarrier allocation, because of the QoS analyzes, resulting in a fairer utilization of the OFDM symbol.

Figure 4. Subcarrier and bit allocation using Two step Algorithm.

3.3. Modified Water-filling Algorithm

The aim of the MWA is to maximize the system throughput with a limited available power, but considering the minimal data rate of each user as a constraint. The basic idea consists on apply the Water-filling Theorem to find the best subcarriers for each user, based on the minimal data rate required by the users and the individual channel frequency response.

In order to demonstrate the principle of this proposal and its performance, two analyses will be made: i) analyze of the subcarrier distribution and modulation allocation for each user and; ii) analyze of the maximum throughput of the system.

3.3.1. Subcarrier distribution and modulation allocation analysis

In this approach, first, the total available power is divided between the subcarriers using the result presented in (5). Once the available power for a subcarrier is defined, then it is possible to define the largest number of bits per symbol that can be transmitted in this subcarrier by using the result presented in (2). This procedure is evaluated for all users, which means that, at this point, the maximum throughput that an user can achieve, if it could use all subcarriers, is known for all users. The second step consists on distributing the subcarriers between the users, according with the individual required QoS. The subcarrier distribution procedure is performed according with a priority that is based on the QoS of each user. In this algorithm, the user that requires the largest data rate has higher priority than the user that requires a lower data rate. The priority user receives one subcarrier, which is the one that can carry the largest number of bits per symbol. This subcarrier cannot be allocated to any other user. The number of bits per symbol that the allocated subcarrier can transmit is subtracted from the desired data rate of this user. Following, a new priority user is defined and the process continues until all users have their minimal data rate guaranteed. After this, three different approaches can be used to distribute the remaining subcarriers:

a) the distribution of the subcarriers between the users is random, following the uniform distribution – this approach is called Random Priority Distribution;

b) an unused subcarrier is allocated to the user that can transmit the largest number of bits per symbol in this specific subcarrier – this approach is called Maximum Modulation Order Priority;

c) an unused subcarrier is allocated to the user that can achieve the highest average data rate – this approach is called Maximum Rate Priority.

From Figure 5, it is possible to conclude that Random Priority Approach results in a balanced distribution between all users. In this simulation, User 1 has received 87 subcarriers while Users 2 and 3 have received, respectively, 83 and 86 subcarriers. From Figure 6 it is possible to conclude that Maximum Modulation Order Priority benefits the user with best channel condition at the frequency of the unused subcarrier. This means that one user that receives the n<sup>th</sup> subcarrier is not necessary the priority user for the (n+1)<sup>th</sup> subcarrier. The priority analysis is performed on a subcarrier to subcarrier basis. In this simulation, Users 1, 2 and 3 have received, respectively 42, 103 and 111 subcarriers.

From Figure 7, it is possible to conclude that the Maximum Rate Priority approach results in the most unbalanced subcarrier distribution. This outcome is expected because in this case the user with the best average channel condition will always receive a higher priority. In this simulation, Users 1, 2 and 3 have received, respectively, 40, 97 and 119 subcarriers. Obviously, User 3 has the best channel condition while User 1 has the worse channel conditions.

The MWA has presented a more fairer subcarrier and bit allocation when compared with WA because MWA considers the minimal data rate of each user as a constraint.
initial constraint. Depending on the priority approach used with the MWA, it is possible to achieve the same degree of fairness than the TSA, but MWA has the advantage to be simpler than TSA, once TSA does not perform the bit allocation. TSA requires that another algorithm, to allocate the bits per symbol at each distributed subcarrier [5].

3.3.2. Maximum throughput analysis

It is clear from Figure 5, Figure 6 and Figure 7 that the system throughput obtained with MWA varies according with the priority approach used. The aim of this analysis is to verify the spectral efficiency of the three approaches.

All the channels are frequency selective, with channel discrete impulse response given by Table 1 and time variant with Rayleigh distribution. It has been considered that the channel coherence time is larger than the duration of one OFDM symbol, which means that the channels remain time invariant during the transmission of an OFDM symbol. The average data throughput has been evaluated after the transmission of 50 OFDM symbols.

Figure 8 depicts the number of bits allocated for each user per OFDM symbol, as well as the total number of bits transmitted per OFDM symbol, considering the three priority approaches presented in this paper. It is possible to notice that the random priority results in more balanced distribution between the users, although achieves the lowest overall throughput. The maximum rate priority and the maximum modulation order priority present almost the same overall throughput, but with maximum rate priority the individual throughput presents a high variance from one OFDM symbol to other. The maximum modulation order results in a lower individual throughput variation from one OFDM symbol to other.

Figure 9 shows the average number of bits transmitted in the subcarriers of the OFDM symbols per user, as well as the total average number of bits transmitted per each subcarrier of the OFDM symbols. Once again, it is possible to conclude that random priority approach achieves the smallest spectral efficiency, while maximum modulation order priority approach results in the larger spectral efficiency.

Table 2 presents the average number of bits transmitted by each user per OFDM symbol considering the three priority approaches. The total number of bits transmitted per OFDM symbol is also presented in Table 2. Once again, it is possible to conclude that MWA with Maximum Modulation Order Priority achieves the highest total data rate. MWA with Random Priority achieves the lowest total data rate, although an user with bad channel conditions can obtain a higher data rate with this solution when compared with Maximum Modulation Order or Maximum Rate approaches. In Table 3, it is presented the average spectrum efficiency of each priority approach, allowing one to conclude that Maximum Modulation Order Priority results in the highest spectrum efficiency. As expected, the Random Priority approaches results in a lower spectral efficiency.
Figure 8. Individual and total number of bits in an OFDM symbol.

Figure 9. Individual and total average number of bit per subcarrier of the OFDM symbols.
Table 2. Average number of bits transmitted by each user and total number of bits transmitted per OFDM symbol for each priority approach.

<table>
<thead>
<tr>
<th></th>
<th>Bits User 1</th>
<th>Bits User 2</th>
<th>Bits User 3</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Random</td>
<td>300.38</td>
<td>313.06</td>
<td>369.62</td>
<td>327.67</td>
</tr>
<tr>
<td>Maximum Modulation</td>
<td>293.10</td>
<td>318.74</td>
<td>453.56</td>
<td>355.10</td>
</tr>
<tr>
<td>Maximum Rate</td>
<td>263.18</td>
<td>299.76</td>
<td>495.2</td>
<td>352.71</td>
</tr>
</tbody>
</table>

Table 3. Average spectral efficiency of the priorities approaches.

<table>
<thead>
<tr>
<th></th>
<th>Bits/carrier User 1</th>
<th>Bits/carrier User 2</th>
<th>Bits/carrier User 3</th>
<th>Average Bits/carrier</th>
</tr>
</thead>
<tbody>
<tr>
<td>Random</td>
<td>3.601</td>
<td>4.058</td>
<td>4.508</td>
<td>4.056</td>
</tr>
<tr>
<td>Maximum Modulation</td>
<td>3.768</td>
<td>4.067</td>
<td>4.537</td>
<td>4.124</td>
</tr>
<tr>
<td>Maximum Rate</td>
<td>3.808</td>
<td>4.090</td>
<td>4.457</td>
<td>4.118</td>
</tr>
</tbody>
</table>

4. Conclusions

The spectral efficiency of OFDMA system deeply relies on the efficiency of the subcarrier and bit allocation algorithm that is used to distribute the system resource between the users. If the algorithm uses only the individual channel conditions, unfair resource distribution may take place. Thus, it is important to consider the minimal data rate that each user requests in order to guarantee a fair distribution of the resources. The proposed Modified Water-filling Algorithm uses the channel conditions and the user’s QoS to define the number of subcarriers for each user, the position of such subcarriers in the OFDM symbol and the number of bits per symbol that can be transmitted in each subcarrier, considering the target symbol error rate. This solution shows to result in fairly distributed resources between all users. When the number of available subcarriers is larger than the number of requested subcarriers, the extra subcarriers can be distributed among the users. Between the three priorities approaches considered in this paper, the Maximum Modulation Order Priority has presented the highest throughput and, consequently, the highest spectrum efficiency. The Random Priority approach does not consider the channel conditions to distribute the remaining subcarriers and, as a consequence, it achieves the lowest performance. Based on the simulation results presented in this paper, it is possible to conclude that the Modified Water-filling Algorithm with Maximum Modulation Order Priority approach is a simple and suitable solution to perform subcarrier and bit allocation in an OFDMA system.

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