Traffic Optimization for Multimodal Cooperative Networks

Krishna Jyothi Boppana and Daryl Reynolds

Lane Department of Computer Science and Electrical Engineering
West Virginia University, Morgantown, WV 26505-5523
Email: Kboppana@mix.wvu.edu

Abstract—In this paper, we study the traffic optimization problem for a heterogeneous traffic network by considering utility functions of data and voice traffic. We also study the performance of multimodal networks, where other modes of communication like wires, infrared links may operate in addition to wireless nodes to improve the performance of the wireless network. We assume the presence of an additional wired channel between the wireless nodes in an example network. We solve the traffic optimization problem by maximizing the sum of utilities of all voice and data users in the network and calculate the optimal rates that can be allocated to data and voice. We also divide transmit power between the wireless channel and the wired channel for a source-destination pair wireless network and a cooperative diamond relay network, implemented with Laneman protocol separately using constrained optimization techniques and performance is analyzed in terms of outage probability expressions. We show that optimal power allocation to a wired channel between the source and a relay separated by a poor wireless channel improves performance with few assumptions on the characteristics of the wire and over a range of transmit powers.

1. INTRODUCTION

It is challenging to design a network which accommodates diverse applications and different kinds of traffic. High data rates are mainly due to recent innovations in signal processing, modulation techniques and smart antennas that can directly transmit signal to users. Resource allocation and traffic optimization are crucial problems in multi-traffic networks as resources are scarce and traffic is shared by multiple users. In recent years, a lot of research is being focused on the resource allocation problem in multi-traffic communication networks. The distribution of available rate to data packets in a network is a difficult problem as rate, reliability and delay requirements vary for different kinds of traffic. Voice and video are sensitive to delay while a delay-tolerant data service needs sufficient throughput. Traffic can be placed in a queue and rate may be allocated based on first come first serve basis. This is not a viable solution in some cases where some delay sensitive traffic may be held in queue while low priority data packets are being transmitted which increases delay thus decreasing efficiency of the system. Priority queuing [1] is one sophisticated scheme for allocating resources to the traffic with different QoS requirements. In [2], a new scheme, batch and priority based admission control is proposed for multi-rate wireless systems to allocate bandwidth according to various QoS requirements of the traffic. Voice and data can be integrated using some fixed or adaptive boundaries that allocate data and voice in slots based on traffic requirement in integrated voice and data wireless networks [3] [4]. Efficiency of resource allocation can be measured by the achieved network utility or fairness. Power and rate for wireless systems are allocated by solving a constrained optimization problem to achieve minimum transmit power and maximum rate [5]. In [6], network resource optimization in a relay-assisted cellular networks with heterogeneous QoS requirements is done by deriving two efficient algorithms. Zhou et al. [7][8][9] proposed a resource allocation scheme called revenue maximization to maximize total profit in a multi-cell wireless multimedia network by selling resources to the users. For a whole network, the performance can be evaluated in terms of the degree to which it satisfies the service requirements of users’ applications [10] rather than other system-centric quantities like throughput, outage probability and power etc. When application performance is the key concern in a network, utility could be considered as a reliable metric. Utility and optimization based resource allocation problems have been studied by many researchers. These are helpful to design and control wireless and wire-line networks. Kelly et al. [11] analyzed different rate control algorithms based on utility functions for a network model and introduced the framework of Network Utility Maximization (NUM) that has many applications in network resource allocation algorithms. Using utility functions, traffic optimization problems are formulated into convex or non-convex optimization problems and solved using Lagrange multipliers and Kuhn-Tucker theorem. For rate allocation, Network Utility Maximization (NUM) [11] can consider utility functions of all kinds of traffic and allocates rate to achieve their maximum combined utility. The NUM framework [12] has diverse applications in network rate allocating algorithms, in Internet congestion control protocols and network-fairness characterization. In [13], the best use of rate-reliability characteristics at the physical layer to support different kinds of traffic over a network and maximizing their utilities are studied. In this paper, utility functions are considered to measure the level of users’ satisfaction and total system utility as an optimization objective to solve the traffic optimization problem. In [11], two different classes of rate control algorithms are introduced considering utility functions in terms of rate only. As it does not reflect all the user requirements, utility function in terms of rate, delay and reliability should be considered.
In [13], capacity is allocated to mixture of traffic types, including VoIP, delay sensitive and delay-insensitive data traffic considering appropriate rate-reliability characteristics and maximizing their combined utility in a network with composite links. Traffic optimization model in this research is motivation from the work in [11] and [13] which has the same objective of maximizing aggregate utility.

A. Multimodal Networks

A lot of research has been done on capacity limits of wireless networks under some assumptions on the physics of propagation and some restrictions on the communication strategy employed by the nodes. Gupta et al. [14] analyzed capacity of a wireless network with \( n \) nodes located in a region of area \( 1 \, m^2 \). It can been seen that the rate per user drops to zero as the number of users increases in that fixed area. In [15], it is proven that the capacity limits are due to a degree of freedom limitation related to fundamental electromagnetic laws of physics. From these works it can be concluded that very large high-performance purely wireless networks are not just difficult to design, but are actually impossible to build. In such cases, an additional mode of communication like wire solves the problem and decreases the rate burden on wireless channels. The advantage of wires is that they are non-fading and they do not contribute to the large-network bottleneck that drives throughput to zero. A wireless network with wire between nodes is termed as multimodal network [16].

In multimodal networks, the nodes in addition to wireless transceivers have ability to send or receive wired transmission from other nodes. Here wired and/or wireless modes operate simultaneously to improve the performance of the wireless network. Multimodal networks can be used in some scenarios like cellular systems and wire-infrastructure based ad hoc networks, where base stations are connected by wires and close enough to exchange some wireless energy. They can also be used in electrical power system networks, which currently make use of wireless nodes on poles physically co-located with power lines that can also be used for communication. In our lab, we are currently building prototypes of wireless sensor networks using ultrasonic and NFMR links.

Consider a multimodal relay network as shown in the Figure 1. A wire is placed between the source S and relay \( r_1 \). The addition of the wire can reduce outage probability at relay \( r_1 \). But performance of relay \( r_2 \) decreases as the power allocated to wireless channel is lowered. So, optimal power allocation should be done to wired and wireless channel to achieve best results. Research has been focused on optimal power allocation when two modes of communication that operate simultaneously are considered in relay networks.

In [16], a cooperative diamond wireless relay network with nodes having the capability to communicate through wires is considered. It is a jointly optimized multimodal communication network with wired and wireless modes operating simultaneously. The relay network is implemented with an appropriately modified Laneman space time diversity protocol. High-SNR outage probability expressions are developed for two different network topologies. One is with wire between source and relay and other is with wire between relay and destination. Those expressions are similar to the outage probability expressions for Laneman network and are given below. Outage probability expression for the network with wire between source and relay is given by [16]

\[
P_r(I < R) \approx \left[ \frac{2^{2R} - 1}{2^{SNR/m}} \right] \times \sum_{D(s)} \left[ \frac{1}{1 - P_w} \right]^{m - |D(s)| - 1} \times \lambda_s,d(s) \times \prod_{r \notin D(s)} \lambda_{s,r} \times \prod_{r \in D(s)} \lambda_{r,d(s)} \times A(D(s))(2^{2R} - 1)
\]

(1)

where \( \lambda_{s,d} = \frac{B_w}{mB_w + \frac{B_w}{SNR}} \) for wired relay, \( \hat{R} = R - \frac{1}{2} B_w \log(1 + mB_w/(B_w + SNR)) \) and \( \lambda_{s,r} = \lambda_{s,d} \). Outage probability expression for the network with wire between relay and destination is given by [16]

\[
P_r(I < R) \approx \left[ \frac{1}{2^{SNR/m}} \right]^m \times \lambda_s,d(s) \times \left[ \frac{\lambda_{r,w,d(s)}}{(1 - P_w)} \right] \times \sum_{D(s): r \in D(s)} \left( 2^{2R} - 1 \right)^{|D(s)| + 1} \times \prod_{r \notin D(s)} \lambda_{s,r} \times A(D(s))(2^{2R} - 1)
\]

(2)

\[
+ \left( 2^{2R} - 1 \right)^m \times \lambda_{s,w} \times \sum_{D(s): r \notin D(s)} \lambda_{s,r} \times \prod_{r \notin D(s): r \notin D(s)} \lambda_{s,r} \times A(D(s))(2^{2R} - 1)
\]

(3)

B. Overview

In this paper, we concentrate on resource and power allocation problems for heterogeneous traffic multimodal networks. Two types of traffic, delay-sensitive data and voice are considered and rate allocation is done maximizing their combined utility in the network. Power allocation is done for a source-destination pair multimodal network to achieve...
maximum capacity and minimum outage probability separately. For a cooperative diamond relay multimodal network, high-SNR outage probability expressions are studied and optimal power to be sent on wire is calculated analytically using outage probability expressions. Multi-traffic rate allocation and power allocation for a network are formulated as shown in Figure 2. Delay-sensitive data and voice should be sent efficiently through a wireless channel (fading channel) with additional mode of communication (wire).

The resource and power allocation problems for heterogeneous traffic multimodal networks are solved by dividing those problems as shown in the Figure 3. First, the total rate is divided between delay-sensitive data and voice to achieve maximum utility in the network. We consider a utility function for delay-sensitive data as a function of rate, reliability and delay and the utility function of voice is a function of rate and delay [13]. Traffic optimization problem is then solved by maximizing the sum of utilities of all voice and data users in the network and optimal rates that can be allocated to data and voice are calculated. In the next step, we divide transmit power between the wireless channel and the wired channel for a source-destination pair multimodal network and a cooperative diamond multimodal network implemented with Laneman protocol [17] separately using constrained optimization techniques and performance is analyzed in terms of outage probability expressions. For implementing a cooperative wireless network, we consider two cases: wire place between source and relay. Optimal power allocation to wired and wireless channels is done by analytically minimizing the high-SNR outage probability expressions. In the cooperative diamond relay multimodal network, wire is placed between nodes and signal is transmitted using distributed space time decode and forward protocol [17] to analyze outage probability expressions and minimum transmit power that should be sent through wire is calculated.

The paper is organized as follows. Section 2 contains traffic optimization problem for delay-sensitive data and voice and results are presented. In Section 3, a source-destination pair multimodal network is studied and power allocation is done to achieve maximum instantaneous rate and minimum outage probability in the network. In Section 4, a cooperative wireless relay network with a wire between nodes is considered and outage probability for the network implemented with space-time decode and forward protocol is studied and plotted. Section 5 contains numerical results and Sections 6 concludes with some insights to future work.

2. Traffic Optimization

We consider a heterogeneous network with delay-sensitive data and voice traffic. Assume two sources that transmit voice and delay-sensitive data packets with packet size $K$ bits. We also assume that the network can differentiate voice and data. The term data refers to delay sensitive data and voice. The rate $R$ should be shared by the data and voice traffic such that $R = R_d + R_v$. We assume incoming data and voice packets are stored in separate queues and transmitted in a first-in-first-out (FIFO) fashion. The Kleinrock independence approximation [18] can be applied to a queue where packets need to wait for their transmission. According to the Kleinrock independence approximation, the average delay for the queue as mentioned above can be approximately calculated by assuming that the delays in the queues are independent. Applying Kleinrock independence approximation, data and voice queue can be modeled as a $M/D/1$ queue and delay is calculated accordingly. The average end to end delay for the data source is $\delta_d$ and for the voice source is $\delta_v$ and units for $\delta_v$ and $\delta_d$ are sec/wireless Hz. The end to end delay refers to transmission and queuing delays. $\delta_d = 1 - p_d$ and $\rho_v = 1 - p_v$ are reliabilities for data source and voice source respectively, where $p_d$ and $p_v$ are end-to-end packet error probabilities for the data and voice. Traffic optimization is based on maximizing the sum of utilities of all voice and data users.

A. Utility Function for Data

We consider utility function for delay-sensitive data as a function of rate, reliability and delay and is represented as $U_d(R_d, \rho_d, \delta_d)$. It can be a weighted sum of utility on delay and throughput with the weight $w_d \in [0, 1]$ indicating the relative importance of throughput and delay [13]

$$U_d(R_d, \rho_d, \delta_d) = w_d \frac{R_d \rho_d - (r_d \rho_d)^{\text{min}}}{(r_d \rho_d)^{\text{max}} - (r_d \rho_d)^{\text{min}}} - (1 - w_d) \frac{\delta_d - \delta_d^{\text{min}}}{\delta_d^{\text{max}} - \delta_d^{\text{min}}} \tag{4}$$

The average packet delay, $\delta_d$ is calculated for the data source by assuming the Kleinrock independence approximation and
is given by [1] [18][13]

\[ \delta_d = \frac{K}{2} \left( \frac{1}{R} + \frac{1}{R - R_d} \right) \]  

(5)

where \( K \) is the packet size. The data utility \( U_d(R_d, \rho_d, \delta_d) \) is a function that has relative importance of throughput and delay by placing a weight \( w_d \). After substituting \( \delta_d \) in (4)

\[ U_d(R_d, \rho_d, \delta_d) = \frac{w_d R_d \rho_d - (R_d \rho_d)_{\text{min}}}{(R_d \rho_d)_{\text{max}} - (R_d \rho_d)_{\text{min}}} \times \left( \frac{K}{2} \left( \frac{1}{R} + \frac{1}{R - R_d} \right) - \delta_{d_{\text{min}}} \right) + \frac{R_d \rho_d}{(R_d \rho_d)_{\text{max}} - \delta_{d_{\text{min}}} - \delta_{d_{\text{min}}}}. \]  

(6)

### B. Utility Function for Voice

1. **R-factor:** The Rating factor (R-factor) is the output from the E-model, an International Telecommunication Union (ITU) transmission rating model [19] used to ensure that the users are satisfied with the end-to-end transmission performance of voice. The E-model estimates the conversational quality from mouth to ear as perceived by the user at the receiver side. The rating factor \( R^{fac} \) is composed of [19]

\[ R^{fac} = R_o - I_s - I_d - I_c + A \]  

(7)

where \( R_o \) represents the basic SNR, \( I_s \) represents the combination of all impairments which occur more or less simultaneously with the voice signal, \( I_d \) represents the impairments caused by the delay, \( I_c \) represents impairments caused by low bit rate codecs and \( A \) is the advantage factor that corresponds to the user allowance due to the convenience in using a given technology. The parameters \( R_o, I_s \) and \( I_d \) are subdivided into further specific impairment values. The E-model provides a statistical estimation of quality measures. So, the R-factor for voice can be considered as the utility function for voice. The R-factor and voice quality are directly proportional. If the R-factor is 100, the voice quality is said to be the best. In this work, we consider R-factor after considering the subdivided impairment values of \( R_o, I_s \) and \( I_d \) in equation (7) is given by [13]

\[ R^{fac} = R_o - \alpha_1 \delta_v - \alpha_2 (\delta_v - \alpha_3) H - \beta_1 - \beta_2 \log(1 + \alpha_3 \psi) \]  

(8)

where \( \psi \) is the packet loss percentage for the call and the other parameters \( R_o, \alpha_1, \alpha_2, \alpha_3, \beta_1, \beta_2 \) and \( \beta_3 \) are constants. The permitted values and range of values for these constants are specified in [19] from the E-model and values specified in [13] are considered. \( R_o = 94.2, \alpha_1 = 0.024, \alpha_2 = 0.11, \alpha_3 = 177.3, H = 0, \delta_v < \alpha_3 \) otherwise \( H = 1, \beta_1, \beta_2 \) and \( \beta_3 \) are codec dependent parameters. For G729 codec, these parameters are \( \beta_1 = 12, \beta_2 = 15, \beta_3 = 0.6 \). The packet loss percentage can be substituted in terms of \( \rho_v \) as \( \psi = 100 \rho_v = 100(1 - \rho_v) \). Then the utility function for voice in terms of \( \delta_v \) and \( 1 - \rho_v \) is

\[ U_v(\rho_v, \delta_v) = R_o - \alpha_1 \delta_v - \alpha_2 (\delta_v - \alpha_3) H - \beta_1 - \beta_2 \log(1 + 100 \beta_3 (1 - \rho_v)). \]  

(9)

The average packet delay, \( \delta_v \) is given by [1][18][13]

\[ \delta_v = \frac{K}{2} \left( \frac{1}{R} + \frac{1}{R - R_o} \right) \].  

(10)

The R-factor increases as \( \delta_v \) decreases. Substituting \( \delta_v \) in equation (9) we get

\[ U_v(R_v, \rho_v) = R_o - \alpha_1 \frac{K}{2} \left( \frac{1}{R} + \frac{1}{R - R_o} \right) - \alpha_2 \left( \frac{K}{2} \left( \frac{1}{R} + \frac{1}{R - R_o} \right) - \alpha_3 \right) H - \beta_1 - \beta_2 \log \left( 1 + 100 \beta_3 (1 - \rho_v) \right). \]  

(11)

where \( U_v(R_v, \rho_v) \) is utility function of voice in terms of rate and reliability.

### C. Maximizing Total Utility using Lagrange Multipliers Method

The rate optimization problem for the network is solved by maximizing the sum of utilities of all voice and data users. It can be formulated as

maximize \( U = U_d(R_d, \rho_d, \delta_d) + U_v(R_v, \rho_v) \) subject to \( R_d + R_v = R \).  

Lagrange equation for the rate optimization problem can be written as

\[ \Lambda(R_d, R_v, \lambda) = U_d(R_d, \rho_d, \delta_d) + U_v(R_v, \rho_v) - \lambda (R_d + R_v - R). \]  

(12)

Solving the equation (12) using Lagrange multipliers method and KKT conditions gives the optimal values of \( R_d \) and \( R_v \).

**Theorem 2.1:** The optimal rate allocated to data traffic is the value of \( R_d \) that satisfies the equation

\[ \frac{w_d [\rho_d]}{a} - \frac{1 - w_d}{b} \times \frac{K}{2} \left( \frac{a}{(R_d - R)^2} \right) + \frac{K \alpha_1}{2} \times \frac{1}{R_d^2} + \frac{\alpha_3 HK}{2 R_d^2} = 0, \]  

(13)

**Proof:** Refer Appendix A for the proof.

In this work rate allocation is done to achieve maximum utility in the whole network. In order to achieve fairness for data and voice users, we can add minimum achievable utility constraints for voice and data traffic while solving optimization problem. By adding this, minimum required utility can be guaranteed to both data and voice users separately.

### 3. Power Allocation for a Source-Destination Pair Multimodal Network

As discussed in Section 1, the throughput problem cannot be alleviated by any combination of coding, modulation, or networking protocol [15]. So, we can add an additional mode of communication like wire in a wireless network to reduce the total load on wireless channel. We have also seen that power allocation is a crucial part for multimodal networks. Even if we connect the nodes that use wires to a power source, power allocation must be done to both modes as nodes connected to a power source will have infinite energy
The wireless channel is attenuated by a fading coefficient $h_{wl}$ and wireless channels that operate simultaneously. The noise spectral density of the wired channel, $N_w$, is the signal to noise ratio of wired channel and $B$ is the bandwidth, $h_{wl}$ is the fading coefficient, $N_{wl}$ is the noise spectral density and $\text{SNR}_{wl} = \frac{P_{wl}}{N_{wl}B_{wl}}$ is the signal to noise ratio of the wireless channel. The fading coefficient $h_{wl}$ varies for the wireless channel.

The total instantaneous capacity for the multimodal network is the sum of channel capacity of wired channel and instantaneous capacity of the wireless channel. Adding equations (14) and (15) yields, [20]

$$C = C_{w} + C_{wl}$$

$$= \frac{B_w}{B} \log_2 \left( 1 + \frac{|h_w|^2 \text{SNR}_w}{Bw} \right) + \log_2 \left( 1 + \frac{|h_{wl}|^2 P_{wl}}{N_{wl}B_{wl}} \right)$$

where $C$ is total instantaneous rate and units of $C$ are bits/sec/wireless Hz. To achieve maximum rate in the network, the total rate in equation (16) must be maximum. So, the rate optimization problem can be formulated as

$$\text{maximize } C = C_{w} + C_{wl}$$

subject to $P_w + P_{wl} = P$.

The Lagrange equation for the rate optimization problem can be written as

$$\Lambda(P_w, P_{wl}, \lambda) = \frac{B_w}{B} \log_2 \left( 1 + \frac{|h_w|^2 P_w}{N_wB_w} \right) + \log_2 \left( 1 + \frac{|h_{wl}|^2 P_{wl}}{N_{wl}B_{wl}} \right) + \lambda (P_w + P_{wl} - P).$$

Solving the equation (17) using Langrange multipliers method and applying KKT conditions gives the optimal values of $R_d$ and $R_w$.

**Theorem 3.1:** Optimal values of $P_w$ and $P_{wl}$ to achieve maximum instantaneous rate in source-destination multimodal network are

$$P_w = \frac{B_w B_{wl} N_{wl} |h_w|^2 + |h_{wl}|^2 (B_w |h_w|^2 P - B_w B_{wl} N_w)}{|h_{wl}|^2 |h_w|^2 (B_w + B_{wl})}$$

$$P_{wl} = P - P_w.$$  \hspace{1cm} (19)

**Proof:** Refer Appendix B for the proof.

**C. Outage Probability Minimization**

The outage probability for a fading channel is given by

$$P_{out}(R) \approx \frac{2^R - 1}{\text{SNR}} \text{ at high SNR.}$$  \hspace{1cm} (20)

Now, outage probability for the multimodal network shown in Figure 4 is given by [20]

$$P_{out}(R, \text{SNR}_{wl}) = \text{Prob}(C < R)$$

$$= \text{Prob}(C_{wl} < R - C_w)$$

$$= 1 - e^{-\frac{2^{R-C_w} - 1}{\text{SNR}_{wl}}}$$ \hspace{1cm} (21)

$$\approx 2^{R-C_w} - 1 \text{ at high SNR}$$

$$\approx \frac{R - B_w \log_2 \left( 1 + \frac{|h_{wl}|^2 P_{wl}}{N_{wl}B_{wl}} \right)}{2} - 1 \text{ (22)}$$
The equation (22) is to be minimized w.r.t constraint \( P_w + P_{wl} = P \). Outage probability optimization problem is formulated as

\[
\text{minimize } P_{\text{out}}(R, \text{SNR}) = \frac{2}{P_{wl}} \frac{R - B_w \log_2 \left( 1 + \frac{|h_w|^2 P_w}{N_w B_w} \right)}{1 + \frac{|h_w|^2 P_w}{N_w B_w}} \quad (24)
\]

subject to \( P_w + P_{wl} = P \). (24)

The Lagrange equation for the outage minimization problem can be written as

\[
\Lambda(P_w, P_{wl}, \lambda) = \frac{2}{P_{wl}} \frac{R - B_w \log_2 \left( 1 + \frac{|h_w|^2 P_w}{N_w B_w} \right)}{1 + \frac{|h_w|^2 P_w}{N_w B_w}} - 1 + \lambda(P_w + P_{wl} - P). \quad (25)
\]

Minimizing the equation (25) using Langrange multipliers method gives the optimal values of \( P_w \) and \( P_{wl} \).

**Theorem 3.2:** Optimal values of \( P_w \) and \( P_{wl} \) are the values that satisfies the simultaneous equations

\[
\left( R - \left( \frac{B_w}{B_{wl}} \right) \log_2 \left( 1 + \frac{|h_w|^2 P_w}{N_w B_w} \right) \right) \left( \frac{N_{wl}}{P_{wl}} \right) = \frac{|h_w|^2 B_w}{N_w B_w + |h_w|^2 P_w} \quad (26)
\]

\[
+ \lambda_1 = 0 \quad (27)
\]

\[
\left( R - \left( \frac{B_w}{B_{wl}} \right) \log_2 \left( 1 + \frac{|h_w|^2 P_w}{N_w B_w} \right) \right) - 1 \left( \frac{N_{wl} B_w}{P_{wl}^2} - \frac{1}{P_{wl}^2} \right) + \lambda_1 = 0. \quad (28)
\]

**Proof:** Refer Appendix C for the proof.

4. **Outage Probability Minimization in a Cooperative Multimodal Network**

In Section 3, we have optimized instantaneous rate and outage probability for a source-destination pair multimodal network. In [17], the authors analyzed repetition-based and space-time coded cooperative diversity for cooperative wireless relay networks using outage probability as a performance measure. In this section, a diamond multimodal cooperative network, where relays have capability to communicate via wired and wireless channels as previously discussed in Section 1 is considered and optimal power allocation is done analytically using high-SNR outage probability expressions from [16]. The outage probability for the network is considered in two different network topologies.

- Wire placed between source and relay
- Wire placed between relay and destination

In this section some results from [16] are duplicated and analyzed. So the network and outage probability expressions from [16] are considered here.

**A. Cooperative Network Model**

In this paper, we considered a cooperative diamond relay network as shown in Figure 5. The source \( s \) broadcasts the information to the destination \( d(s) \). Two relays \( r_1 \) and \( r_2 \) account to the cooperating terminals together with the destination \( d(s) \). \( r_w \) is used for the relay where wire is attached. Channel gains \( a_{s,r_a}, a_{s,d(s)} \) are between source and relay \( r_a \) and source and destination, \( a_{r_a,d(s)} \) between relays and destination. \( 1/\lambda_{s,r_a}, 1/\lambda_{s,d(s)}, 1/\lambda_{r_a,d(s)} \) are variances for corresponding channel gains. Channel gains are considered to be constant during the transmission. As decode and forward cooperative diversity protocol is considered, the decoding set \( D(s) \) can be \( \{\Phi\}, \{r_1\}, \{r_2\} \) or \( \{r_1, r_2\} \).

Figures 5 and 6 describe the network topologies with wire placed between source-relay and relay-destination respectively. Assume additional mode (wire) has bandwidth \( B_{wl} \). Channel gain \( h \) is also assumed to be constant across the wired channel. Let \( B_{wl} \) be the bandwidth of the wireless channel. In space-time coded cooperative diversity, the terminals transmit in half the available degrees of freedom, so the SNR in the network is normalized \( \frac{2}{N_w} \text{SNR} \). Power is divided between two channels, \( P_w \) is the percentage of power assigned to the additional mode which varies between 0 and 1. As wire is attached to relay \( r_1, r_1=r_w \) is used.

1The decoding set \( D(s) \) is the set of relays that can decode the message after every transmission
B. Outage Probability for the Network with Wire between Source and Relay

In this section, outage probability for the first network topology as shown in the Figure 5 is studied. Let \( \lambda_{s;r_d} = \lambda_{s;r} \). For wired relay, redefining \( \lambda_{s;r} = \frac{\lambda_{s;r_d}}{2^{SR - 1}} \)

where \( \hat{R} = R - \frac{B_{sw}}{2 R_{sw} \log \left( 1 + \frac{2 P_w}{m B_{sw}/B_{snr}[\text{SNR}]} \right) } \). At high-SNR, the information outage probability is given as [16]

\[
P_s[I < R] \approx \frac{2^{2R-1}}{2^{SNR/m}} \times \sum_{D(s)} \left[ \frac{1}{1 - P_w} m - |D(s)| - 1 \right] 
\times \lambda_{s,d,s} \prod_{r \in D(s)} \lambda_{s,r} \prod_{r \in D(s)} \lambda_{r,d(s)} 
\times A_{D(s)}(2^{2R} - 1) \tag{26}
\]

where \( A_n(t) = \frac{1}{(n-1)!} \int_0^1 \frac{e^{-w(n-1)(1+tw)}}{1+tw} \, dw \), \( n > 0 \), and \( A_0 = 1 \).

The expression for the information outage probability for the network with wire between source-relay for the network shown in Figure 5 from the equation (26) is given by [16]

\[
P_s[I < R] \approx \frac{2^{2R-1}}{2^{SNR/m}} \times \lambda_{s,d,s} \left( \frac{1}{1 - R_{sw}} \right)^2 \times \lambda_{s,r_w} 
\times \lambda_{s,d,s} \left( 1 + \frac{P_{sw}}{m B_{sw}/B_{snr}[\text{SNR}]} \right) \times \lambda_{s,r_1} 
+ \left( \frac{1}{1 - P_w} \right) \times \lambda_{s,r_w} \lambda_{s,d,s} \times \lambda_{s,r_1} 
\times A_1(2^{2R} - 1) \times \lambda_{r_d(s)} \times A_2(2^{2R} - 1) 
\times A_{r_d(s)} \times A_{D(s)}(2^{2R} - 1) \tag{27}
\]

**Theorem 4.1:** The optimal power for the wired channel is the value of \( P_w \) that satisfies the equation

\[
\begin{align*}
\left( 2 \lambda_{s,r_1} + \frac{2 P_{sw}}{m B_{sw}/B_{snr}[\text{SNR}]} \right) 
\times \lambda_{s,r} 
\times \lambda_{r_d(s)} 
\times A_1(2^{2R} - 1) 
\times A_{r_d(s)} \times A_{D(s)}(2^{2R} - 1) 
\end{align*}
\]

**Proof:** Refer Appendix D for the proof.

C. Outage Probability for the Network with Wire between Relay and Destination

When wire is placed between \( r_w \) and \( d(s) \) as shown in Figure 6, if the relay decodes the message from the source, then the outage probability expression for the network with wire between relay and destination is given by [16]

\[
P_s[I < R] \approx \frac{1}{2^{SNR/m}} \times \lambda_{s,d,s} \times \sum_{D(s)} \left[ \frac{1}{1 - P_w} (2^{2R} - 1)^{|D(s)|+1} \right] 
\times \lambda_{r_d(s)} 
\times \prod_{r \in D(s)} \lambda_{r_d(s)} 
\times \prod_{r \notin D(s)} \lambda_{r_d(s)} 
\times A_{D(s)}(2^{2R} - 1) \tag{28}
\]

The outage probability expression for the network in Figure 6 with wire between relay and destination is [16]

\[
P_s[I < R] = \frac{1}{2^{SNR/m}} \times \lambda_{r_w,d(s)} \times \left[ \frac{1}{1 - P_w} \times \lambda_{r_d(s)} \times A_1(2^{2R} - 1) \times A_2(2^{2R} - 1) 
\times \lambda_{r_d(s)} \times A_1(2^{2R} - 1) \right] \tag{29}
\]

**Theorem 4.2:** The optimal power that can be allocated to wired channel is the value of \( P_w \) that satisfies the equation

\[
\begin{align*}
(2t_1 \times \frac{dt_1}{dP_w}) \times t \times \lambda_{s,r_2} \times f_1 
+ \left( \frac{dt_1}{dP_w} \times \lambda_{s,r_2} \times \frac{dt_1}{dP_w} \right) 
\times \lambda_{s,r_2} \times f_1 
\end{align*}
\]

**Proof:** Refer Appendix D for the proof.

5. NUMERICAL RESULTS

Figure 7 provide numerical results for the network described in the Section 3.1. We consider packet size \( K = 5 \); packet loss, \( p_d = 10^{-6} \) and \( p_w = 10^{-3} \); data parameters \( r_{d_{\text{max}}} = R, r_{d_{\text{min}}} = 0.1, \delta_{d_1} = 0 \) sec/wireless Hz, \( \delta_{d_{\text{max}}} = 0.2 \) sec/wireless Hz and weight \( w_d = 0.7 \). Total utility, the sum of utilities of voice and data is calculated for three different values of \( R \), then data allocation is done at the point where maximum total utility occurs as shown in the Figure 7. When sufficient amount of rate is allocated to data and voice, total utility almost becomes constant and maximum total utility is achieved at the point where delay for the traffic is minimum. Figure 8 shows power optimization problem results for the network maximum instantaneous capacity scenario for the network shown in Figure 4. Outage
probability vs wired power plots are drawn for different bandwidth ratios of wired and wireless channels. Pathloss and fading effects are considered and pathloss exponent value of 3 is used for calculations. Total instantaneous rate and optimal wired power are more when $B_w = 10 \times B_{wl}$. When channel bandwidths are equal, optimal wired power and wireless power are almost equal. When $B_w = \frac{1}{10} B_{wl}$, total instantaneous capacity is less compared to former cases and optimal wireless power is more than optimal wired power. As capacity depends on the value of $h$, in Figure 9, the total instantaneous rate and average of instantaneous rate vs wired power are plotted for different values of $h$. The maximum average instantaneous rate point for $B_w = B_{wl}$ case is also plotted in the Figure 9.

Figures 10, 11 and 12 show the results for power optimization problem to achieve minimum outage probability for the network shown in the Figure 4 for different ratios of wired and wireless channel bandwidths. Exact and approximate values of outage probability for different ratios of wired and wireless channel bandwidths are plotted using equations (21) and (22) respectively. Outage probability with optimal power values is plotted by varying the $\text{SNR}_{wl}$. From the plots, it is observed that the outage probability with optimal power values is less than the outage with equal power values. More gain in outage probability is observed when $B_w = 10 \times B_{wl}$ which can be seen in Figure 12.
power allocation is done at the minimum outage probability point where the wired channel can support at least rate $R$. From the plots it is observed that when $P_w$ increases, the outage probability decreases as the power for other relay decreases reducing its decoding capability. Network performance is better when wire is placed between source-relay as outage probability for wire between source-relay case is less compared to relay-destination case.

6. CONCLUSION

Instantaneous rate maximization and outage probability minimization are proposed for optimal power allocation in source-destination pair multimodal networks. For cooperative multimodal networks, analytical minimization of outage probability is proposed for optimal power allocation to wired and wireless modes.

The performance of source-destination pair multimodal network is observed to be better when wired channel bandwidth is greater than wireless channel bandwidth. In such case, maximum instantaneous rate and minimum outage probability are observed after optimal power allocation. So, we can conclude that rate and outage probability optimizations improve the performance of the network. In a cooperative multimodal network, outage probability is studied for a wide range of input power and optimal power values for both the modes are calculated.

REFERENCES

A. Proof for Maximizing Total Utility in Data and Voice Networks

The Lagrange equation for solving rate optimization problem after substituting utility functions of data and voice is given by

$$\Lambda(R_d, R_v, \lambda) = U_d(R_d, \rho_d, \delta_d) + U_v(\rho_v, R_v) + \lambda(R_d + R_v - R)$$

$$= \frac{w_d R_d \rho_d - (R_d \rho_d)^{\min}}{R_d \rho_d^m a} - \frac{w_d R_d \rho_d - (R_d \rho_d)^{\min}}{R_d \rho_d^m b}$$

$$+ \lambda(R_d + R_v - R)$$

Let $a = (R_d \rho_d)^{\max} - (R_d \rho_d)^{\min}$ and $b = (\delta_d^{\max} - \delta_d^{\min})$. Substituting the values of $a$ and $b$ in (A-19) we get

$$\frac{\partial \Lambda}{\partial R_d} = \frac{w_d \rho_d}{a} - \frac{1 - w_d}{{\alpha_1}} \frac{1}{(R - R_d)^2} + \lambda$$

Differentiating the equation (31) w.r.t $R_d$ we get

$$\frac{\partial \Lambda}{\partial R_v} = -\frac{K}{2} \alpha_1 \left( \frac{1}{(R - R_v)^2} - \frac{H \alpha_1 K}{2} \frac{1}{(R - R_v)^2} \right) + \lambda$$

Using KKT conditions we can equate equations (32) and (33) to zero. Applying KKT condition to equation (33), we will get the value of $\lambda$ as

$$\lambda = \frac{1}{2} \alpha_1 \frac{1}{R_d^2} + \frac{\alpha_2 H K}{2} \times \frac{1}{R_d^2}$$

Substituting the value of $\lambda$ from (34) and $R_v = R - R_d$ in (32) and equating to zero we get

$$\frac{w_d \rho_d}{a} - \frac{1 - w_d}{b} \times \frac{K}{2} \left[ \frac{a}{(R - R_d)^2} \right] + \frac{K \alpha_1}{2} \times \frac{1}{R_d^2} \times \frac{\alpha_2 H K}{2R_d^2} = 0.$$
in matlab gives the optimal value of \( R_d \) and \( R_v = R - R_d \) gives the optimal value of \( R_v \).

**B. Proof for Outage Minimization in a Source-Destination Pair Multimodal Network**

The Lagrange equation for the capacity optimization problem is formulated as

\[
\Lambda(P_w, P_{wl}, \lambda) = \frac{B_w}{B_{wl}} \log_2 \left( 1 + |h_w|^2 \frac{P_w}{N_w B_w} \right) + \log_2 \left( 1 + |h_w|^2 \frac{P_{wl}}{N_{wl} B_{wl}} \right) + \lambda (P_w + P_{wl} - P).
\]

Differentiating equation (36) partially w.r.t \( P_w \)

\[
\frac{\partial \Lambda}{\partial P_w} = B_w \left( \frac{1}{1 + |h_w|^2 \frac{P_w}{N_w B_w}} \right) \left( \frac{|h_w|^2}{N_w B_w} \right) + \lambda.
\]

Differentiating equation (36) partially w.r.t \( P_{wl} \)

\[
\frac{\partial \Lambda}{\partial P_{wl}} = B_{wl} \left( \frac{1}{1 + |h_w|^2 \frac{P_{wl}}{N_{wl} B_{wl}}} \right) \left( |h_w|^2 \frac{P_{wl}}{N_{wl} B_{wl}} \right) + \lambda.
\]

Differentiating equation (36) partially w.r.t \( \lambda \) and equating to zero

\[
\frac{\partial \Lambda}{\partial \lambda} = P_w + P_{wl} - P = 0.
\]

Using KKT conditions we can equate (37) and (38) to zero. Equating (36) to zero we will get the value of \( P_w \)

\[
\lambda = B_w \left( \frac{1}{1 + |h_w|^2 \frac{P_w}{N_w B_w}} \right) \left( \frac{|h_w|^2}{N_w B_w} \right).
\]

Substituting the value of \( \lambda \) in (38) \((B_w|h_w|^2)(N_{wl}B_{wl} + |h_w|^2(P - P_w)) - (|h_w|^2B_{wl})(N_{wl}B_{wl} + |h_w|^2P_w) = 0\)

Value of \( P_w \) form the above equation is,

\[
P_w = \frac{N_{wl}B_{wl}B_{wl}|h_w|^2 + B_w|h_w|^2|h_w|^2P - |h_w|^2B_{wl}B_{wl}N_w}{|h_w|^2|h_w|^2(B_w + B_{wl})}.
\]

So, wireless power \( P_{wl} \) is

\[
P_{wl} = P - P_w.
\]

**C. Proof for Outage Minimization in a Source-Destination Pair Multimodal Network**

The Lagrange equation for this problem from equation (25) is given by

\[
\Lambda_1(P_w, P_{wl}, \lambda_1) = \frac{2 - (B_w/B_{wl}) \log \left( 1 + \frac{|h_w|^2 P_w}{N_w |h_w|^2} \right)}{N_w B_{wl}} - 1 + \lambda_1 (P_w + P_{wl} - P).
\]

Differentiating the equation (43) with respect to \( P_w \) and equating to zero we get

\[
\frac{\partial \Lambda_1}{\partial P_w} = 2 - (B_w/B_{wl}) \log \left( 1 + \frac{|h_w|^2 P_w}{N_w |h_w|^2} \right) \left( \frac{N_{wl}}{P_{wl}} \right) - \frac{|h_w|^2B_w}{(N_w B_{wl} + |h_w|^2 P_w)} + \lambda_1 = 0.
\]

Differentiating the equation (43) with respect to \( P_{wl} \) and equating to zero we get

\[
\frac{\partial \Lambda_1}{\partial P_{wl}} = \left( \frac{2 - (B_w/B_{wl}) \log \left( 1 + \frac{|h_w|^2 P_{wl}}{N_{wl} |h_w|^2} \right)}{N_{wl} B_{wl}} - 1 \right) \frac{N_{wl} B_{wl} - 1}{P_{wl}} + \lambda_1 = 0.
\]

Equations (44) and (45) are two simultaneous equations with two unknown variables \( P_w \) and \( P_{wl} \). These two equations are solved in matlab numerically to get the optimal values of \( P_w \) and \( P_{wl} \).

**D. Proofs for Minimizing Outage probability for Cooperative Diamond Relay Multimodal Network**

1) Wire between Source and Relay : The expression for information outage probability for the network with wire between source and relay is [16]

\[
P_r[I < R] \approx \left( 2^{2R - \frac{1}{2} SNR/m} \right) \lambda_s d(s) \left( \frac{1}{\left( 1 - P_w \right)^2} \right) \lambda_s d(s) + \lambda_s r(s) A_1(2^{2R} - 1).
\]

The partial derivative of (47) w.r.t to \( P_w \) is

\[
\frac{\partial P_r}{\partial P_w} = \frac{2^{2R} - 1}{2^{SNR/m}} \lambda_s d(s) \left[ \lambda_s r(s) \right. + \left. \frac{2^{2R - \frac{1}{2} SNR/m} \log \left( 1 + \frac{2^{2R} - 1}{SNR/m} \right)}{1 + \frac{2^{2R - \frac{1}{2} SNR/m} \log \left( 1 + \frac{2^{2R} - 1}{SNR/m} \right)}{2^{SNR/m}}} \right].
\]

The partial derivative of (47) w.r.t to \( P_{wl} \) is

\[
\frac{\partial P_r}{\partial P_{wl}} = \frac{2^{2R} - 1}{2^{SNR/m}} \lambda_s d(s) \left[ \lambda_s r(s) \right. + \left. \frac{2^{2R - \frac{1}{2} SNR/m} \log \left( 1 + \frac{2^{2R} - 1}{SNR/m} \right)}{1 + \frac{2^{2R - \frac{1}{2} SNR/m} \log \left( 1 + \frac{2^{2R} - 1}{SNR/m} \right)}{2^{SNR/m}}} \right].
\]
The optimal $P_w$ value in the equation (47) is found using the numerical values of (48) in matlab.

2) Wire between Relay and Destination: The outage probability expression for the network with wire between relay and destination is [16]

$$P_r(I < R) = \frac{1}{2SNR/m} \times \lambda_{r_w,d(s)} \times \left[ \frac{\lambda_{r_w,d(s)}}{1 - P_w} \left( (2^{2R} - 1)^2 \times (2^{2R} - 1) \times \lambda_{s,r_2} \times A_1(2^{2R} - 1) + (2^{2R} - 1)^3 \times \lambda_{r_2,d(s)} \times A_2(2^{2R} - 1) \right) \right].$$

(49)

Let $t = 2^{2R} - 1, t_1 = 2^{2R} - 1, f = A_1(2^{2R} - 1), f_1 = A_1(2^{2R} - 1), f_2 = A_2(2^{2R} - 1)$. Then the equation (49) can be written as

$$P_r(I < R) = \frac{1}{2SNR/m} \times \lambda_{r_w,d(s)} \times \left[ \frac{\lambda_{r_w,d(s)}}{1 - P_w} \left( t_1^2 \times t \times \lambda_{s,r_2} \times f_1 + t_1^3 \times \lambda_{r_2,d(s)} \times f_2 \right) + (t^3 \times \lambda_{s,r_2} \times (\lambda_{r_2,d(s)} \times f_1)) \right].$$

(50)

Differentiating the equation (50), we get

$$\frac{\partial P_r}{\partial P_w} = \frac{\lambda_{r_w,d(s)}}{1 - P_w} \left[ 2t_1 \times \frac{dt_1}{dP_w} \times t \times \lambda_{s,r_2} \times f_1 + t_1^2 \times t \times \lambda_{s,r_2} \times f_1 + t_1^3 \times \lambda_{r_2,d(s)} \right] \times \frac{df_1}{dP_w} + 3t_1^2 \times \frac{dt_1}{dP_w} \times \lambda_{r_2,d(s)} \times f_2.$$}

(51)

Numerical values of (51) are calculated to get the optimal value of $P_w$ for the equation (49) in matlab.