Abstract—This paper presents the theoretical analysis of the acquisition phase of a chaos-based DS-CDMA system in the presence of fast fading. A statistical approach is taken to accurately model the acquisition phase with fast fading. Using this statistical approach, the main performance criteria for the acquisition phase namely probabilities of detection and failure are derived. The analytical derivation results are then compared to the simulation results with close agreement. It is found that the model developed is accurate in describing the behavior and performance of the acquisition phase in the presence of fast fading.

Index Terms—acquisition, fast fading, chaos-based communications

I. INTRODUCTION

EVER since their introduction to digital communications, chaotic spreading codes have been a subject for active research. The reason for such a high interest stems from them satisfying the three aspects of secret communication set out by Shannon which are concealment, privacy and encryption in physical layer [1]–[3]. Moreover, the hypersensitivity of chaotic codes to their initial condition is utilized to generate unique orthogonal spreading codes which can be used for multi-user chaos-based DS-CDMA communication systems [2], [4], [5].

Many studies into the suitability of chaotic codes for spreading purposes in DS-CDMA have been carried out and it was found in [3], [6]–[11] that suitably chosen chaotic codes can be used in robust multi-user DS-CDMA systems. Chaotic codes are also shown to improve the system performance compared to the classical Pseudo Random Binary Sequences (PRBS), Gold codes and $m$ sequences [12], [13]. Also, in the context of the maximum error free transmission rate, it is shown in [3], [14], [15] that chaotic codes perform better than classical spreading codes.

A DS-CDMA system requires that the transmitter and receiver spreading waveforms be synchronized. If the two waveforms are out of synchronization by as much as one chip duration, the amount of energy reaching the receiver will be proportional to the cross-correlation value which is not sufficient for reliable message detection [16], [17].

Chaos-based DS-CDMA systems are no exception to the synchronization requirement and synchronizing chaotic codes has been an active area of research ever since their introduction to communications. The synchronization analysis for conventional spreading codes cannot be applied to chaotic codes because they are non-binary, that is they can assume any value between two set thresholds. Many papers including [18], [19] have shown that robust acquisition of chaos-DS-CDMA systems is possible. Moreover, [20] has given a complete analysis of a non-coherent tracking scheme for chaos-based system with verified results on a DSP. Recently [21] has given the complete analysis of chaos-based acquisition and tracking in presence of fading coefficients which are constant over a bit period. However, the models proposed have a limited use when the fading coefficients change with a higher rate especially with fading scenarios which include fast movement of scattering objects [22].

Many papers including [23]–[25] have addressed the code acquisition problem in fading conditions, but these do not cover the situations in which the fading coefficients change faster than the bits. Also, nearly all the publications only assume binary spreading codes for acquisition analysis, therefore there is a need to address the performance of chaos-based code acquisition in fast fading conditions. In this contribution, we present a new and complete analysis of chaotic spreading code acquisition in presence of fast fading. In order to determine the performance, the classical analysis of acquisition performance had to be abandoned in favor of a new statistical method. Analytical results have been derived for acquisition performance and are verified by their close agreement with the simulation results.

The rest of this paper is structured as follows. Section II presents the theoretical analysis of the acquisition phase under fast fading conditions. Section III presents the simulation results and their comparison with the analytical ones and finally Section IV presents the conclusions.

II. THEORETICAL ANALYSIS OF CODE ACQUISITION IN THE PRESENCE OF FAST FADING

This section presents the theoretical background related to chaos-based spreading codes followed by the proposed statistical model and treatment of the acquisition phase under fast fading conditions.

A. Theoretical background of chaos-based spreading codes

Chaotic spreading codes are ideally non-periodic, random looking, orthogonal and wide-band and are generated from discrete chaotic maps [26]. The orthogonality of the chaotic spreading codes is shown in Fig. 1. The logistic map (a subset of the Chebychev chaotic map) is used for this analysis. Chebychev maps are iterative and in case of the logistic map,
the generating equation is \( x_{k+1} = 2x_k^2 - 1 \) [26]. Given that the initial condition for the chaotic map can be any number within \((-1, 1)\) except -0.5, 0.5 and 0, taking all the possible initial conditions into account can only be achieved by assuming the chaotic value to be a random variable with a certain Probability Density Function (PDF). The Chebychev maps have invariant probability density [26] therefore, the results can be easily extended to any other subset of the Chebychev map.

The PDF of the logistic map is [26]:

\[
P_x(x) = \begin{cases} 
\frac{1}{\pi\sqrt{x^2 - 1}} & -1 < x < 1 \\
0 & \text{otherwise} 
\end{cases}.
\] (1)

The mean and variance values can be easily calculated as 0 and \(\frac{1}{2}\) respectively.

**B. Statistical analysis of code acquisition in presence of fast fading**

The code acquisition system in chaos-based DS-CDMA uses the same serial search algorithm which is the popular solution to the code acquisition problem in most of Direct Sequence Spread Spectrum (DS-SS) systems. The algorithm is based on dividing the propagation delay between the transmitter and receiver into smaller time periods, each of which are as long as a chip duration. Then a locally generated pilot code, delayed by the estimate of the relative delay between the transmitter and receiver, is continuously correlated with the incoming periodic pilot code. If the correlation result is above a certain threshold, synchronization is declared and the delay estimate will be passed on to the user code generators to de-spread the messages properly. If the correlation value is below the threshold, then the delay estimate will be shifted by one chip length and the correlation between received and locally generated pilots will be repeated.

The issues with the classical treatment of such a system are twofold. First, there is no treatment of the case for which the fading coefficients change from chip to chip (fast fading). Second, the non-binary nature of the spreading code examined here has to be taken into account. Since the classic treatment of the acquisition phase does not provide for these two issues, it is abandoned in favor a different approach.

In order to prove the concept, a base-band code acquisition system is examined in this paper. Fig. 2 shows the block diagram of the base-band code acquisition system used in the analysis. As can be seen a multiplier and summer form a correlator in the forward loop. For ease of understanding, time \(t\) is assumed to be in discrete steps of one chip duration.

\[
r_t = s_t + n_t,
\] (2)

where \(n_t\) is the additive white Gaussian noise with the power \(\sigma_n^2\) and \(s_t\) is the transmitted signal which can be expressed as

\[
s_t = \alpha_{t-\tau} x_{t-\tau} \gamma_{t-\tau}
\] (3)

where \(x_{t-\tau}\) is the transmitted chaotic pilot code which is delayed by \(\tau\), \(\gamma_{t-\tau}\) is the pilot modulating signal which is assumed to equal 1 for all \(t\) and \(\alpha_{t-\tau}\) is the fast fading coefficient which changes for every chip. The fading coefficient follows a Rayleigh distribution expressed as

\[
f_\alpha (\alpha) = \frac{\alpha}{b^2} \exp \left( -\frac{\alpha^2}{2b^2} \right),
\] (4)

where \(b\) is the mode of the Rayleigh distribution and is chosen such that \(E[\alpha^2] = 1\).
The correlation is performed on a per chip basis. For a certain correlation length \((L)\) all the transmitter and locally generated pilot chips are multiplied together and summed for that correlation length. The correlation length is a fraction of the pilot period. The result of the correlation can be expressed as

\[
Z_j = \sum_{t=0}^{L} r_t x_{t-j}
\]

\[
= \sum_{t=0}^{L} \alpha_{t-\tau} x_{t-\tau} x_{t-j} + \sum_{t=0}^{L} n_t x_{t-j}, \quad (5)
\]

where \(Z_j\) is the \(j^{th}\) index of the correlation output.

It is evident that in general there could be two distinct cases for \(Z_j\) as \(j\) changes. If \(j = \tau\) then the transmitter and receiver codes are aligned and \(Z_{j=\tau}\) is reduced to

\[
Z_{j=\tau} = \sum_{t=0}^{L} \alpha_{t-\tau} x_{t}^2 + \sum_{t=0}^{L} n_t x_{t-\tau}, \quad (6)
\]

which is the faded auto-correlation function of the chaotic spreading code.

Since the problem of synchronization is inherently statistical, the correlation function has to be statistically examined. Given that all possible values of the chaotic spreading codes can be assumed to be a random variable, and \(\alpha\) is also a random variable, it is possible to propose a PDF for \(Z_{j=\tau}\). It is evident that \(Z_{j=\tau}\) is a summation of \(L\) identically distributed random variables where \(L \gg 1\). Given each of these random variables are being generated using independent experiments, they are also independent. In this case no matter what the distribution of \(\alpha_{t-\tau} x_{t-\tau}^2\) is, it can be assumed to be Gaussian through the invocation of the Central Limit Theorem. A similar approach can be used for \(n_t x_{t-j}\).

The mean and variance of the resulting Gaussian distributed random variable can be easily calculated as

\[
E[Z_{j=\tau}] = \frac{Lb}{2} \sqrt{\frac{\pi}{2}},
\]

\[
\sigma^2_{Z_{j=\tau}} = L \left\{ \frac{b^2 (6 - \pi) + 4 \sigma_n^2}{8} \right\}. \quad (8)
\]

The correlation function distribution for \(j \neq \tau\) can be expressed in a similar way but with a different mean and variance. It can be easily shown that

\[
E[Z_{j\neq\tau}] = 0,
\]

\[
\sigma^2_{Z_{j\neq\tau}} = L \left\{ \frac{b^2 + \sigma_n^2}{2} \right\}. \quad (10)
\]

The correlation function \(Z_j\) is statistically defined, however the acquisition phase is concerned with the square of the correlation function. Therefore, the PDFs of \(Z_j^2\) have to be defined where \(Z_j^2 = \frac{Z_j^2}{\sigma^2_j}\).

Since \(Z_j^2\) is the square of \(Z_j\) which has a non-zero mean Gaussian distribution, it follows a non-central chi-square distribution with one degree of freedom. This distribution can be expressed as

\[
f_{Z_j^2}(Z_j^2) = \frac{1}{2} \exp - \left( Z_j^2 + \lambda \right) \left( \frac{Z_j^2}{\lambda} \right)^{-1} I_{\frac{1}{2}} (\sqrt{\lambda Z_j}) \cdot \quad (11)
\]

where

\[
\lambda = \frac{L\pi}{(6 - \pi) + 4 \sigma_n^2}, \quad (12)
\]

is the non-centrality parameter.

Similarly the PDF of \(Z_{j\neq\tau}^2\) can be defined as a central chi-square distribution with one degree of freedom given by

\[
f_{Z_j^2}(Z_j^2) = \frac{1}{\sqrt{2\Gamma(\frac{1}{2})}} Z_j^{\frac{1}{2}} \exp - \left( \frac{Z_j^2}{2} \right). \quad (13)
\]

Now that the decision variable \(Z_j\) has been statistically described, it is possible to evaluate the performance of the acquisition phase. The acquisition phase performance can be analyzed using two probabilities. The first is the probability of detection \((p_D)\), which is the probability that the decision variable \(Z_j\) is larger than the pre-set threshold \(Z_{th}\) when \(j = \tau\). If such event happens then synchronization is declared correctly and the system can commence the de-spreading of the message. The next probability that has to be evaluated is the false alarm probability \((p_F)\), which is the probability that \(Z_{th}\) is exceeded when \(j \neq \tau\). Clearly this probability has to be minimized as it will cause loss of data.

To acquire these two probabilities, the expressions given in (11) and (13) have to be integrated across a suitable region. Therefore \(p_D\) can be expressed as

\[
\int_{Z_{th}}^{\infty} f_{Z_j^2}(Z_j^2) dZ_j^2 = 1 - \frac{\text{erf} \left( \sqrt{\frac{Z_{th}^2}{2}} - \sqrt{\frac{\lambda}{2}} \right)}{\sqrt{2}} + \frac{\text{erf} \left( \sqrt{\frac{Z_{th}^2}{2}} + \sqrt{\frac{\lambda}{2}} \right)}{\sqrt{2}}. \quad (14)
\]

Similarly \(p_F\) can be evaluated as

\[
\int_{Z_{th}}^{\infty} f_{Z_j^2}(Z_j^2) dZ_j^2 = 1 - \text{erf} \left( \sqrt{\frac{Z_{th}^2}{2}} \right). \quad (15)
\]

Equations given in (14) and (15) present the analytical expressions of the acquisition phase performance. They show the probabilities of detection and failure when the acquisition is performed under fast fading conditions. These expressions will be validated through comparison with the simulation results in the next section.
III. RESULTS AND COMPARISON

The acquisition phase has been simulated for different correlation lengths at different signal to noise ratios. The pilot length is 1000 chips. The signal to noise ratio is calculated based on the energy per one bit of the pilot. The system is assumed to have a spreading factor of 100. As mentioned in the previous section, the fading mode $b$ is chosen such that $E[\alpha^2] = 1$. The simulations have $10^4$ trials each. For each trial the correlation vector is formed with different noise and fading coefficients, $p_D$ and $p_F$ are subsequently calculated by the number of threshold crossings for each $Z_{th}$ starting from 0 to the maximum possible $Z_{th}$.

Figures 3 (a) and (b) show $p_D$ plotted against $p_F$. Clearly both probabilities change between 0 and 1. The figures are known as the Receiver Operating Characteristic (ROC) in literature and they give a meaningful indication of the acquisition phase performance. The goal is to have the highest possible $p_D$ for the lowest possible $p_F$. Therefore the ROC curves that are more to the top left indicate better synchronization performance.

Fig. 3 (a) shows the ROC for 2 dB at 200 and 400 chip correlation lengths ($L$) respectively. As can be seen the simulation and analytical results have close agreement. Also the 400 chip correlation result is higher than the 200 chip one. This is due to the correlator receiving more information which results in a higher auto-correlation peak. This however is at the cost of more processing time for the incoming information.

The same trend is repeated in Fig 3 (b). As the signal to noise ratio increases the acquisition phase performance improves as expected. The agreement between the simulation and analytical results remains close.

IV. CONCLUSIONS

A chaos-based DS-CDMA acquisition system has been investigated under fast fading conditions. A new way of describing the behavior of the system and evaluating its performance has been provided which is based on the statistics of the non-binary chaotic spreading codes used in the system. The close agreement of analytical and simulation results suggests that the acquisition phase can be accurately described using the proposed method. The effect of correlation length on acquisition performance is described analytically.

REFERENCES


