Filter Bank Multicarrier for Next Generation of Communication Systems

Behrouz Farhang-Boroujeny

Electrical and Computer Engineering Department

University of Utah

Emails: farhang@ece.utah.edu

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Organization (1/2)

- **Introduction**
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  - Primary and Secondary Users (PUs and SUs)
  - Spectrum sensing and sharing
  - Spectrum leakage
  - Multiple access networks

- **FFT-based OFDM**
  - Subcarrier leakage
    - Solutions and limitations

- **Spectrum analysis methods**
  - Periodogram Spectral Estimator (PSE)
  - Blackman-Tukey Spectral Estimator (BTSE)
  - Minimum Variance Spectral Estimator (MVSE)
  - Multitaper Method (MTM)
  - Filter Bank Spectral Estimator (FBSE)
Organization (2/2)

- Filterbank as multicarrier communication tools
  - Filtered multitone (FMT)
  - Offset QAM/Staggered modulated multitone (SMT)
  - Cosine modulated multitone (CMT)

- Prototype Filter Design
  - Design for time-invariant channels
  - Design for doubly dispersive channels

- Implementation of Filterbank Multicarrier Systems
  - Polyphase structures

- Conclusions
Introduction: What is Cognitive Radio?

Cognitive Radios:

- Have the capability to be aware of their surrounding environment
- Can change PHY depending on environment
- Can change PHY depending on traffic needs
- Can alter higher layer behavior as needed
- Learn from past experiences


Capable of complex adaptation on lower layers
Introduction: Primary and secondary users

- Primary (licensed) and secondary (unlicensed) users coexist and share the same spectrum
- PUs have priority and thus SUs must back-off as soon as PUs begin a communication
  - This requires channel sensing
Introduction

Multicarrier has been proposed for channel sensing and co-existence of PUs and SUs

Introduction

Multicarrier has been proposed for channel sensing and co-existence of PUs and SUs

To avoid interference among primary and secondary users good separation/filtering of different subcarriers is necessary.

Multicarrier Methods:

Conventional OFDM
FFT-based OFDM: Transceiver Structure

TRANSMITTER

Serial input

S/P

Encoder

IFFT + CP

P/S

Channel

RECEIVER

Serial output

P/S

Decoder + FEQ

Remove CP + FFT

S/P

S/P: Serial-to-Parallel

P/S: Parallel-to-Serial

CP: Cyclic Prefix

TEQ: Time Domain Equalizer

FEQ: Frequency Domain Equalizer
FFT-based OFDM: Transceiver Structure

**TRANSMITTER**

- Serial input
- **S/P**
- Encoder
- IFFT + CP
- **P/S**

**RECEIVER**

- Serial output
- **P/S**
- Decoder + FEQ
- Remove CP + FFT
- **S/P**

Bandwidth loss incurs because of CP

S/P: Serial-to-Parallel
P/S: Parallel-to-Serial
CP: Cyclic Prefix
TEQ: Time Domain Equalizer
FEQ: Frequency Domain Equalizer
FFT-based OFDM: Parameters definition

$N$: The maximum number of subcarriers / FFT length

$C$: The number of cyclic prefix samples

$T_s$: The sample interval (in sec.)

$T = NT_s$: The duration of each FFT block

$T_G = CT_s$: The duration of each cyclic prefix / guard interval

$T_S = T + T_G$: The duration of each OFDM symbol

$f_i$: The center frequency of the $i$th subcarrier

$X_i$: The data symbol at the $i$th subcarrier (i.e., in freq. domain)

$g(n)$: The symbol shaping window ($n = 0, 1, \ldots, N+C-1$)

$x_i(n) = X_i g(n) e^{j2\pi (n-C-1) i / N}$: The $i$th subcarrier signal samples in time domain (before modulation to RF band); $n$ is time index
FFT-based OFDM: Transmit signal and its spectrum

Power spectrum of the $i$th subcarrier:

$$\Phi_{xix} (f) = K |G(f - f_i)|^2$$

Adding up power spectra of all active subcarriers, the power spectrum of transmit signal $x(t)$ is obtained as:

$$\Phi_{xx} (f) = \sum_{i} \Phi_{xixi} (f)$$

FFT-based OFDM: Transmit signal and its spectrum

In conventional OFDM, where $g(n) = 1$, for $n = 0, 1, \ldots, N+C-1$,

$$\Phi_{x_i x_i}(f) = K |\text{sinc}((f - f_i)T_s)|^2, \text{ where } \text{sinc}(x) = \frac{\sin(\pi x)}{\pi x}$$

- Poor side lobes
  - interference with PUs
- First side lobe is at -13 dB!
FFT-based OFDM: Improving the spectrum of transmit signal

• The large side lobes in the PSD of each subcarrier is a direct consequence of using a rectangular window.
• The side lobes can be suppressed significantly by using a window that rolls-off gently.
• This increases the duration of each OFDM symbol from $T_S$ to $(1+\beta)T_S$

➤ Hence, further loss in bandwidth efficiency.
**FFT-based OFDM: Improving the spectrum of transmit signal**

- Side lobes are suppressed by increasing $\beta$
- To suppress side lobes sufficiently, $\beta$ values of 0.5 or greater may be required (Wiess et al. (2004))

> Hence, significant loss in bandwidth efficiency

**Notes:**

- The side lobes adjacent to the main lobe remain significant, even for $\beta = 1$
- To solve this problem one may introduced guard bands between PU and SU bands

> Hence, further loss in bandwidth efficiency
FFT-based OFDM: Interference introduced by PUs and other SUs on a SU

- Traditional OFDM applies FFT to a length N, CP-removed (rectangular) window, of the received signal.
  - This is equivalent to applying a bank of bandpass filters with modulated sinc frequency responses.
  - The large side-lobes of the sinc responses result in significant energy pick up from the bands that are unsynchronized with the intended bands.

  ➢ Hence, significant interference will be picked up.

- Solution: apply a window function with gentle transition to zero, before applying FFT.

  ➢ Further reduction in bandwidth efficiency
FFT-based OFDM: Reducing interference from PUs and other SUs on a SU

- Filtering is performed on \((1+\alpha)N\) samples and the result is decimated to \(N\) samples.
  - This is achieved by performing aliasing in time domain (as shown above) and then applying an \(N\)-point FFT.

FFT-based OFDM: SUMMARY

Advantages

• OFDM is a well-studied method.
• OFDM chip-sets are already developed/available.
• Perfect cancellation of ISI and ICI is achieved, thanks to CP.

Disadvantages

• Hard to synchronize when subcarriers are shared among different transmitters.
• Small asynchronicity between different transmitters results in significant intercarrier interference.
• In cognitive radio, significant overhead should be added to avoid interference between primary and secondary users.
Spectral Estimation
Spectral Estimation Methods: Parametric spectral estimation

The signal \( x(n) \) whose spectrum is desired is treated as a random process and modeled as in the following figure.

- The input \( \nu(n) \) is a white random process with variance of unity.
- The parameters \( a_k \) and \( b_k \) are optimized such that \( x(n) \) and \( \hat{x}(n) \) have the closet autocorrelation coefficients.

\[
H(z) = \frac{\sum_{k=0}^{N-1} b_k z^{-k}}{\sum_{k=0}^{M-1} a_k z^{-k}}
\]

\[
\Phi_{xx}(f) = \Phi_{\nu\nu}(f) \left| H(e^{j2\pi f}) \right|^2
\]

\[
= \left| H(e^{j2\pi f}) \right|^2
\]
Spectral Estimation Methods: Non-parametric spectral estimation

Different types of non-parametric spectral estimators:

- Periodogram Spectral Estimator (PSE)
- Blackman-Tukey Spectral Estimator (BTSE)
- Minimum Variance Spectral Estimator (MVSE)
- Maltitaper Method (MTM)
- Filter Bank Spectral Estimator (FBSE)

Spectral Estimation Methods: Non-parametric spectral estimation

Periodogram Spectral Estimator (PSE): obtains the spectrum of a random process $x(n)$, based on the observed samples $\{x(n), x(n-1), x(n-2), \ldots, x(n-N+1)\}$, by evaluating the amplitude of the DFT of the observed vector

$$x(n) = \begin{bmatrix}
x(n) \\
x(n-1) \\
x(n-2) \\
\vdots \\
x(n-N+1)
\end{bmatrix}$$
Spectral Estimation Methods: Non-parametric spectral estimation

**PSE as a filterbank:** The process of applying IDFT to the observed vector \( x(n) \), may be also formulated as

\[
\hat{\Phi}_{\text{PSE}}(f) = \left| \sum_{k=0}^{N-1} h(k) x(n - k) \right|^2
\]

where

\[
h(k) = \frac{1}{\sqrt{N}} e^{j2\pi f k}
\]

For IDFT we have a bank of filters centered at

\[
f = 0, \frac{1}{N}, \frac{2}{N}, \ldots, \frac{N-1}{N}
\]
Spectral Estimation Methods: Non-parametric spectral estimation

DFT filters:

\[ |H(f)| = \frac{1}{\sqrt{N}} \left| \frac{\sin(N\pi(f - f_i))}{\sin(\pi(f - f_i))} \right| \approx \sqrt{N} \left| \text{sinc}(N(f - f_i)) \right| \]

Relatively large side-lobes result in significant leakage of spectral power among different bands.

- Hence, reduces the spectral dynamic range.
Spectral Estimation Methods: Non-parametric spectral estimation

Resolution of the estimates of PSD in PSE:

\[ x(n) \xrightarrow{h(n)} y(n) \xrightarrow{|.|^2} \hat{\Phi}_{\text{PSE}}(f) \]

Since \( x(n) \) is a random process, the filter output \( y(n) \) is a random variable.

Moreover, \( y(n) \), in general, can be approximated by a Gaussian, since it is constructed by linearly combining (a large) set of samples of \( x(n) \). Accordingly, \( |y(n)|^2 \) has a chi-square distribution with 2 degree of freedom, viz.,

\[ \hat{\Phi}_{\text{PSE}}(f) \sim \chi^2_2 \]

Thus,

\[ \text{VAR}[\hat{\Phi}_{\text{PSE}}(f)] = 2E^2[\hat{\Phi}_{\text{PSE}}(f)] \]

Because of their large variance, the PSD estimates are not reliable.
Spectral Estimation Methods: Non-parametric spectral estimation

Prototype filter of the DFT filterbank:

- The prototype filter in DFT has the coefficient vector

\[ h_0(n) = \frac{1}{\sqrt{N}}, \quad \text{for } n = 0, 1, \ldots, N - 1 \]

- The ith-band filter in DFT has the coefficient vector

\[ h_i(n) = h_0(n)e^{j2\pi in/N} = \frac{1}{\sqrt{N}}e^{j2\pi in/N} \]

- The prototype filter is a lowpass filter and the ith-band filter is obtained by modulating it.

- The prototype filter is also the 0th-band filter in the filterbank.
Spectral Estimation Methods: Non-parametric spectral estimation

Matrix formulation the DFT filterbank:

\[
\begin{bmatrix}
X_n(0) \\
X_n(1) \\
X_n(2) \\
\vdots \\
X_n(N-1)
\end{bmatrix} = \begin{bmatrix}
1 & 1 & 1 & \cdots & 1 \\
1 & e^{j2\pi/N} & e^{j4\pi/N} & \cdots & e^{j2\pi(N-1)/N} \\
1 & e^{j4\pi/N} & e^{j8\pi/N} & \cdots & e^{j2\pi(N-1)/N} \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
1 & e^{j2\pi(N-1)/N} & e^{j4\pi(N-1)/N} & \cdots & e^{j2\pi(N-1)^2/N}
\end{bmatrix}
\]

May be viewed as a rectangular window/prototype filter coefficients that are applied to input samples before taking the Fourier transform.
Spectral Estimation Methods: Non-parametric spectral estimation

A snapshot of PSE:

Notes:

• DFT size $N = 256$.
• The expected large variance of the estimates is seen.
• Small spectral dynamic range of PSE is a direct consequence of large side lobes in DFT filters.
Spectral Estimation Methods: Non-parametric spectral estimation

Blackman-Tukey Spectral Estimator (BTSE):

- The large side lobes in PSE arises because of the use of a poor prototype filter.
  - The problem can be resolved by improving the prototype filter, equivalent to applying a window function before filtering.
  - Window function can be directly applied to the signal samples or to the auto-correlation coefficients of the signal samples.
  - When window function is applied to the auto-correlation coefficients of the signal samples, the resulting method called BTSE.
Spectral Estimation Methods: Non-parametric spectral estimation

Blackman-Tukey Spectral Estimator (BTSE): Examples of common windows (length input signal sample = M+1, N = 2M+1)

- Rectangular:
  \[ w(k) = \begin{cases} 
  \frac{1}{\sqrt{2M+1}}, & |k| \leq M \\
  0, & |k| > M 
\end{cases} \]
  \[ W(\omega) = W_{\text{rect}}(\omega) = \frac{\sin \frac{\omega}{2} (2M+1)}{\sqrt{2M+1} \sin \frac{\omega}{2}} \]

- Hanning:
  \[ w(k) = \begin{cases} 
  \frac{1}{2\sqrt{2M+1}} (1 + \cos \frac{\pi k}{M}), & |k| \leq M \\
  0, & |k| > M 
\end{cases} \]
  \[ W(\omega) = \frac{1}{4} W_{\text{rect}}(\omega - \frac{\pi}{M}) + \frac{1}{2} W_{\text{rect}}(\omega) + \frac{1}{4} W_{\text{rect}}(\omega + \frac{\pi}{M}) \]

- Hamming:
  \[ w(k) = \begin{cases} 
  \frac{1}{\sqrt{2M+1}} (0.54 + 0.46 \cos \frac{\pi k}{M}), & |k| \leq M \\
  0, & |k| > M 
\end{cases} \]
  \[ W(\omega) = 0.23 W_{\text{rect}}(\omega - \frac{\pi}{M}) + 0.54 W_{\text{rect}}(\omega) + 0.23 W_{\text{rect}}(\omega + \frac{\pi}{M}) \]
Spectral Estimation Methods: Non-parametric spectral estimation

Blackman-Tukey Spectral Estimator (BTSE): Examples of common windows

Note: side lobes have decreased at the cost of wider main lobe.

- Reduction of Leakage among different bands, i.e., increased spectral dynamic range, is traded at the cost of a lower resolution in frequency.
Spectral Estimation Methods: Non-parametric spectral estimation

Minimum Variance Spectral Estimator (MVSE):

- Each point of PSD is estimated using a different filter.

- These filters are adopted to the spectrum whose estimate is desired.

- Each filter is selected to have a gain of unity at the center of the passband, while the side lobes are optimized for minimum leakage of energy from other bands.

- We do not explore this method as a good candidate for spectrum sensing in CR, mostly because of its computational complexity.
Spectral Estimation Methods: Non-parametric spectral estimation

Multitaper Method (MTM):

- This method replaces the single (prototype) filter in the previous methods by a few filters for measurement of each point of PSD.
- All filters have the same passband. However, by design they are orthogonal, hence, their outputs are a set of uncorrelated random variables.
- The output power of the filters are averaged to reduce the variance of the estimates.
- A set of prototype filters are used for all the bands and polyphase architecture is used for efficient implementation.
- The prototype filters are a set of prolate filters that satisfy some desirable properties/optimality conditions, as discussed in the next slide.

Spectral Estimation Methods: Non-parametric spectral estimation

Origin of the Prolate Filters: *Slepian Sequences*

- The Slepian sequences $s_k = [s_k(1), s_k(2), \ldots, s_k(N)]^T$, $k=1, 2, \ldots, K$ constitute a set of $K$ unit length orthogonal bases vectors which are used to obtain an optimal expansion of the time sequence $x(n) = [x(n-N+1), x(n-N+2), \ldots, x(n-1), x(n)]^T$ over the frequency band $[f_i - \Delta f/2, f_i + \Delta f/2]$.

- The expansion of $x(n)$ has the form of

  $$x(n) = \kappa_1 s_1 + \kappa_2 s_2 + \ldots + \kappa_K s_K,$$

  where

  $$\kappa_k = s_k^H x(n)$$

- To maximize the accuracy of the estimate, the Slepian sequences are chosen such that their spectrum is maximally concentrated over the desired band $[f_i - \Delta f/2, f_i + \Delta f/2]$. This can be related to the **minimax theorem**.
Spectral Estimation Methods: Non-parametric spectral estimation

Minimax Theorem:

The distinct eigenvalues $\lambda_1 > \lambda_2 > \ldots > \lambda_N$ of the correlation matrix $R$ of an observation vector $x(n)$, and their corresponding eigenvectors, $q_1, q_2, \ldots, q_N$, may be obtained through the following optimization procedure:

$$\lambda_1 = \max E[|q_1^H x(n)|^2], \text{ subject to } q_1^H q_1 = 1$$

and for $k = 2, 3, \ldots, N$

$$\lambda_k = \max E[|q_k^H x(n)|^2], \text{ subject to } q_k^H q_k = 1 \text{ and } q_k^H q_i = 0, \text{ for } i=1,2,\ldots,k-1$$

Alternatively, the following procedure may also be used:

$$\lambda_N = \min E[|q_N^H x(n)|^2], \text{ subject to } q_N^H q_N = 1$$

and for $k = N-1,\ldots, 2,1$

$$\lambda_k = \min E[|q_k^H x(n)|^2], \text{ subject to } q_k^H q_k = 1 \text{ and } q_k^H q_i = 0, \text{ for } i=N,N-1,\ldots,k+1$$

Spectral Estimation Methods: Non-parametric spectral estimation

Prolate Filters Design:

1. Construct the correlation matrix $\mathbf{R}$ of a random process with the power spectral density

$$
\Phi_{xx}(f) = \begin{cases} 
1, & |f| < \frac{\Delta f}{2} \\
0, & \text{otherwise}
\end{cases}
$$

$\mathbf{R}$ is a Toeplitz matrix with the first row of

$$
[\phi(0) \phi(1) \cdots, \phi(N - 1)], \text{ with } \phi(n) = \Delta f \text{sinc}(\Delta fn)
$$

2. The first $K$ eigenfilters corresponding to the largest eigenvalues of $\mathbf{R}$ are the coefficient vectors of the prolate filters.
Spectral Estimation Methods: Non-parametric spectral estimation

An Example of Prolate Filters:

Filter parameters:
- Number of subbands: \( N=16 \)
- Filter length: \( L=8N=128 \)

Observations:
- Only the first few filters have good stopband attenuation.

\[ \text{Hence, to have a good spectral dynamic range, only the outputs of the first few prolate filters can be used.} \]
Spectral Estimation Methods: Non-parametric spectral estimation

An Example of Spectrum Sensing Using Prolate Filters:

Filter parameters:
- Number of subbands: $N=128$
- Filter length: $L=8N=1024$

Observation:
- As predicted, to have a good spectral dynamic range (say, 60 dB or better), not more than three prolate filters should be used.
Spectral Estimation Methods: Non-parametric spectral estimation

Adaptive MTM: Thompson proposed the following formulae in order to reduce the impact of poor side lobes of higher numbered prolate filters:

\[
\hat{S}(f) = \frac{\sum_{k=0}^{K-1} |d_k(f)|^2 \hat{S}_k(f)}{\sum_{k=0}^{K-1} |d_k(f)|^2}
\]
and

\[
d_k(f) = \frac{\sqrt{\lambda_k S(f)}}{\lambda_k S(f) + B_k(f)}
\]

Leakage power from other bands

Starting with coarse estimate of \(S(f)\), \(d_k(f)\) is estimated and used to improve the estimate of \(S(f)\). Iterations continue until \(S(f)\) converges.

A very complex procedure!

Spectral Estimation Methods: Non-parametric spectral estimation

An Sample result of adaptive MTM:

- Results are average of 10,000 snapshots.
- Vertical lines indicate 95% confidence intervals.
- Variances are larger at lower levels of PSD.
Spectral Estimation Methods: Non-parametric spectral estimation

An Sample result of adaptive MTM (continued):

Effective Degree of Freedom (EDF):

\[ v(f) = 2 \sum_{k=0}^{K-1} |d_k(f)|^2 \]

When all the averaging factors \(|d_k(f)|^2\) are equal, there are \(K\) complex Gaussian random variables (\(S_k(f)\)’s) whose magnitudes are averaged. This results in a chi-square random variable with 2\(K\) degrees of freedom: \(v(f) = 2K\).

When a few of the factors \(|d_k(f)|^2\) are zero, the degree of the freedom decreases accordingly, and thus the variance of the estimated spectrum, \(S(f)\), increases.
Spectral Estimation Methods: Non-parametric spectral estimation

An Sample result of adaptive MTM (continued):

Effective Degree of Freedom (EDF) for the PSD results presented earlier.
Spectral Estimation Methods: Non-parametric spectral estimation

An Sample result of adaptive MTM (continued):

The 95% boundary limits of a chi-square distribution with various degrees of freedom.

![Graph showing the 95% boundary limits of a chi-square distribution with various degrees of freedom.](image-url)
Spectral Estimation Methods: Non-parametric spectral estimation

Filter Bank Spectral Estimator (FBSE):

\[ x(n) \xrightarrow{H(f)} H(f) \xrightarrow{\text{square and average}} \Phi_{\text{FBSE}}(0) \]

\[ x(n) \xrightarrow{H(f - f_1)} H(f - f_1) \xrightarrow{\text{square and average}} \Phi_{\text{FBSE}}(f_1) \]

\[ \vdots \]

\[ x(n) \xrightarrow{H(f - f_{N-1})} H(f - f_{N-1}) \xrightarrow{\text{square and average}} \Phi_{\text{FBSE}}(f_{N-1}) \]

Notes:

- \( H(f) \), known as prototype filter, is centered around \( f = 0 \).
- The rest of the filters are frequency-shifted/modulated copies of \( H(f) \).

Spectral Estimation Methods: Non-parametric spectral estimation

An Example of Spectrum Sensing Using FBSE:

Magnitude responses of three bandpass filters in a filter bank

Snapshots of FBSE and other techniques

Filter parameters:
- Number of subbands: $N=256$; Filter length: $L=6N=1536$;
- The data length for FBSE is $8N = 2048$; Output power of each subband is measured by averaging over three samples at spacing 256.
- The data lengths for PSE and MTM are 256 and 1024 samples, respectively.

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CONCLUSIONS:

• Periodogram spectral estimation may be insufficient to achieve the required spectral dynamic range.

• The multitaper method gives excellent results, with limited length of data. However, it is too complex to implement.

• Filter bank spectral estimation gives excellent result. Yet, it may cost very little if filter bank multicarrier is adopted for signal modulation.
Filter Bank Multicarrier (FBMC) Methods
Filter Bank Multicarrier (FBMC) Methods

- **Filtered Multitone (FMT):** Uses subcarrier bands with no overlap. Data symbols are quadrature amplitude modulated (QAM).
  - Guard bands are used to separate subcarrier bands. This results in some loss in bandwidth efficiency

- **Multicarrier with Offset QAM/Staggered Modulated Multitone (SMT):** Subcarrier bands are maximally overlapped/minimally spaced.
  - Carrier spacing = symbol rate

- **Cosine Modulated Multitone (CMT):** Uses pulse amplitude modulated (PAM) symbols with vestigial sideband modulation. Subcarrier bands are maximally overlapped / minimally spaced.
  - Carrier spacing = one half of symbol rate

Both SMT and CMT achieve maximum bandwidth efficiency
Filter Bank Multicarrier (FBMC) Methods: FMT (Summary)

- FMT follows the simple principles of the conventional frequency division multiplexing (FDM).
  - Subcarrier bands have no overlap.

- To allow an efficient implementation based on polyphase structures, (i) some restrictions on the position of subcarrier bands are imposed; (ii) a prototype filter is used for all subcarrier bands.
- Transmit symbols, in general, are QAM and $H(f)$ and $H^*(f)$ are a pair of root-Nyquist filters.
- Equalizers are needed after decimators at the receiver.
- A choice of $K > N$ allows addition of guard bands between subcarrier bands.

\[ e^{j2\pi f_0 n} \]

\[ e^{-j2\pi f_0 n} \]

\[ e^{j2\pi f_1 n} \]

\[ e^{-j2\pi f_1 n} \]

\[ e^{j2\pi f_{N-1} n} \]

\[ e^{-j2\pi f_{N-1} n} \]

Transmitter

Receiver


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Filter Bank Multicarrier (FBMC) Methods: SMT (Summary)

- SMT allows overlap of adjacent bands.
  - Maximizes bandwidth efficiency ($K = N$)

- Transmit symbols are offset QAM: in-phase and quadrature components have a time offset of half symbol interval (not shown below), i.e., time staggered.

- If the overlaps are limited to adjacent bands and $H(f)$ and $H^*(f)$ are a pair of root-Nyquist filters, separation of data symbols at the receiver output is guaranteed.

- Equalizers are needed after decimators at the receiver.

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Filter Bank Multicarrier (FBMC) Methods: CMT (Summary)

- CMT allows overlap of adjacent bands.
  - Maximizes bandwidth efficiency \( K = N \)

- Transmit symbols are PAM (pulse amplitude modulated). To allow maximum bandwidth efficiency, vestigial sideband modulation is adopted.

- The overlaps are limited to adjacent bands to simplify filter designs. Here, also, selection of root-Nyquist filters for \( H(f) \) and \( H^*(f) \) guarantees separation of data symbols at the receiver.

- Equalizers are needed after decimators at the receiver.

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Staggered Modulated Multitone (SMT) Details
Filter Bank Multicarrier (FBMC) Methods: SMT (Synthesis/TX)

\[ s_k(t) = \sum_{n=-\infty}^{\infty} s_k[n] \delta(t - nT) \]

\[ s_k[n] = s_k^I[n] + js_k^Q[n] \]
Filter Bank Multicarrier (FBMC) Methods: SMT (Analysis/RX)
Intersymbol Interference (ISI):

- (b) follows from (a) because $h(t)$ is a real-valued function.
- To achieve ISI free transmission $h(t)$ must be a square-root Nyquist and symmetric pulse shape.
Filter Bank Multicarrier (FBMC) Methods: SMT (Details)

Intercarrier Interference (ICI):

The path between the \( k+1 \)th and \( k \)th subcarrier:

\[
\begin{align*}
\mathcal{R}\{\cdot\} &: \quad h(t) \\
\mathcal{I}\{\cdot\} &: \quad h(t - \frac{T}{2}) \\
\end{align*}
\]

Here, the outputs are ICI terms from \( k+1 \)th to the \( k \)th subcarrier. Thus, for ICI free transmission the output samples must be zero.

Equations that show this are presented in the next slide. One of the four different cases is presented. The rest are similar.
The impulse response between the input $s_{k+1}^l(t)$ and the output before the sampler in the upper-right branch of the figure in the previous slide is given by

$$g_1(t) = \Re \left\{ h(t) e^{j\left(\frac{2\pi}{T} t + \frac{\pi}{2}\right)} \right\} \ast h(t) = \left( h(t) \cos \left( \frac{2\pi}{T} t + \frac{\pi}{2} \right) \right) \ast h(t)$$

$$= - \left( h(t) \sin \left( \frac{2\pi}{T} t \right) \right) \ast h(t) = - \int_{-\infty}^{\infty} h(\tau) \sin \left( \frac{2\pi}{T} \tau \right) h(t - \tau) d\tau.$$ 

Substituting $t=nT$, we get

$$g_1(nT) = - \int_{-\infty}^{\infty} h(\tau) \sin \left( \frac{2\pi}{T} \tau \right) h(nT - \tau) d\tau.$$ 

Applying the change of variable $\tau$ to $\frac{nT}{2} + \tau$

$$g_1(nT) = - \int_{-\infty}^{\infty} h \left( \frac{nT}{2} + \tau \right) \sin \left( \frac{2\pi}{T} \tau + n\pi \right) h \left( \frac{nT}{2} - \tau \right) d\tau$$

$$= (-1)^{n+1} \int_{-\infty}^{\infty} h \left( \frac{nT}{2} + \tau \right) h \left( \frac{nT}{2} - \tau \right) \sin \left( \frac{2\pi}{T} \tau \right) d\tau = 0$$

even \quad odd
Filter Bank Multicarrier (FBMC) Methods: SMT (Details)

Channel Impact:

- In general, channel introduces a gain that varies across the channel.
- However, if the number of subcarriers is large, the gain over each subcarrier band may be approximated by a complex-valued constant, say $h_k$.
- When this approximation holds, the channel effect can be compensated for each subcarrier by using a single tap equalizer whose gain is set equal to $1/h_k$.
- Hirosaki (1981), has explored the problem for the more general case where channel gain varies across each subcarrier band and suggested a fractionally-spaced transversal equalizer for each subcarrier.

Cosine Modulated Multitone (CMT) Details
Filter Bank Multicarrier (FBMC) Methods: CMT (Details)

Modulation type: Vestigial Side-Band (VSB)

Data symbols: PAM (Pulse Amplitude Modulation)
Filter Bank Multicarrier (FBMC) Methods: CMT (Details)

Block diagram of a VSB transceiver.

Detailed:

Simplified:
Filter Bank Multicarrier (FBMC) Methods: CMT (Details)

Spectra of various signals in Figure (a) of the previous slide.

Set 1:
Filter Bank Multicarrier (FBMC) Methods: CMT (Details)

Spectra of various signals in Figure (a) of the previous slide.

Set 2:
Filter Bank Multicarrier (FBMC) Methods: CMT (Details)

Intersymbol interference (ISI): Ignoring the channel distortion, the impulse response across a VSB channel is obtained as

\[ g(t) = \Re \{ h(t) e^{j \frac{\pi}{2T} t} \ast h(t) e^{j \frac{\pi}{2T} t} \} \]
\[ = \Re \left\{ \int_{-\infty}^{\infty} h(\tau) e^{j \frac{\pi}{2T} \tau} h(t-\tau) e^{j \frac{\pi}{2T} (t-\tau)} d\tau \right\} \]
\[ = \Re \left\{ e^{j \frac{\pi}{2T} t} \int_{-\infty}^{\infty} h(\tau) h(t-\tau) d\tau \right\} . \]

Letting \( \int_{-\infty}^{\infty} h(\tau) h(t-\tau) d\tau = p(t) \), this simplifies to

\[ g(t) = p(t) \cos \left( \frac{\pi}{2T} t \right) . \]

Thus,

\[ g(nT) = p(nT) \cos \left( \frac{\pi}{2T} nT \right) = p(nT) \cos \left( \frac{n\pi}{2} \right) . \]
Intersymbol interference (ISI): Ignoring the channel distortion, the impulse response across a VSB channel is obtained as

$$g(t) = \Re\{h(t)e^{j\frac{\pi}{2T}t} \ast h(t)e^{j\frac{\pi}{2T}t}\} = \Re\left\{\int_{-\infty}^{\infty} h(\tau)e^{j\frac{\pi}{2T}\tau} h(t-\tau)e^{j\frac{\pi}{2T}(t-\tau)} d\tau\right\} = \Re\left\{e^{j\frac{\pi}{2T}t} \int_{-\infty}^{\infty} h(\tau)h(t-\tau)d\tau\right\}.$$ 

Letting \(\int_{-\infty}^{\infty} h(\tau)h(t-\tau)d\tau = p(t)\), this simplifies to

$$g(t) = p(t) \cos\left(\frac{\pi}{2T}t\right).$$

Thus,

$$g(nT) = p(nT) \cos\left(\frac{\pi}{2T}nT\right) = p(nT) \cos\left(\frac{n\pi}{2}\right).$$

Equal to zero for \(n\) non-zero even integer \hspace{2cm} equal to zero for \(n\) odd
Intersymbol interference (ISI): Ignoring the channel distortion, the impulse response across a VSB channel is obtained as

\[ g(t) = \Re \{ h(t)e^{j\frac{\pi}{2T}t} \ast h(t)e^{j\frac{\pi}{2T}t} \} = \Re \left\{ \int_{-\infty}^{\infty} h(\tau)e^{j\frac{\pi}{2T}\tau} h(t-\tau)e^{j\frac{\pi}{2T}(t-\tau)} d\tau \right\} = \Re \left\{ e^{j\frac{\pi}{2T}t} \int_{-\infty}^{\infty} h(\tau)h(t-\tau)d\tau \right\}. \]

Letting \( \int_{-\infty}^{\infty} h(\tau)h(t-\tau)d\tau = p(t) \), this simplifies to

\[ g(t) = p(t) \cos \left( \frac{\pi}{2T}t \right). \]

Thus,

\[ g(nT) = p(nT) \cos \left( \frac{\pi}{2T}nT \right) = p(nT) \cos \left( \frac{n\pi}{2} \right). \]

Equal to zero for \( n \) non-zero even integer equal to zero for \( n \) odd

(i.e., \( p(t) \) is Nyquist pulse with zero crossing at the interval 2T)
Filter Bank Multicarrier (FBMC) Methods: CMT (Synthesis/TX)
Filter Bank Multicarrier (FBMC) Methods: CMT (Receiver/RX)
Filter Bank Multicarrier (FBMC) Methods: CMT (Details)

The impulse response between the input $s_{k+1}(t)$ at the transmitter and the output before the sampler at the kth subcarrier of the receiver is

$$g_1(t) = \Re\{h(t)e^{j\left(\frac{3\pi}{2T}t + \frac{\pi}{2}\right)} \star h(t)e^{j\frac{\pi}{2T}t}\} = \Re\left\{\int_{-\infty}^{\infty} h(\tau)e^{j\left(\frac{3\pi}{2T}\tau + \frac{\pi}{2}\right)} h(t-\tau)e^{j\frac{\pi}{2T}(t-\tau)} d\tau\right\}
$$

$$= \Re\left\{e^{j\left(\frac{\pi}{2T}t + \frac{\pi}{2}\right)} \int_{-\infty}^{\infty} h(\tau)e^{j\frac{\pi}{2T}\tau} h(t-\tau)d\tau\right\}.$$

Noting that

$$e^{j\left(\frac{\pi}{2T}t + \frac{\pi}{2}\right)} = -\sin\left(\frac{\pi}{2T}t\right) + j\cos\left(\frac{\pi}{2T}t\right)$$

and, since $h(t)$ is a real function,

$$\int_{-\infty}^{\infty} h(\tau)e^{j\frac{\pi}{2T}\tau} h(t-\tau)d\tau = \int_{-\infty}^{\infty} h(\tau)h(t-\tau)\cos\left(\frac{\pi}{T}\tau\right) d\tau + j \int_{-\infty}^{\infty} h(\tau)h(t-\tau)\sin\left(\frac{\pi}{T}\tau\right) d\tau.$$ 

Hence,

$$g_1(t) = -\sin\left(\frac{\pi}{2T}t\right) \int_{-\infty}^{\infty} h(\tau)h(t-\tau)\cos\left(\frac{\pi}{T}\tau\right) d\tau - \cos\left(\frac{\pi}{2T}t\right) \int_{-\infty}^{\infty} h(\tau)h(t-\tau)\sin\left(\frac{\pi}{T}\tau\right) d\tau.$$ 

We are interested in the sample values of $g_1(t)$ at the time instants $nT$, for integer values of $n$. 

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Filter Bank Multicarrier (FBMC) Methods: CMT (Details)

For an even value of $n = 2k$, one finds that $\sin \left( \frac{\pi}{2T} 2kT \right) = \sin(k\pi) = 0$ and, thus

$$g_1(2kT) = (-1)^{k+1} \int_{-\infty}^{\infty} h(\tau)h(2kT - \tau) \sin \left( \frac{\pi}{T}\tau \right) d\tau$$

where we have noted that $\cos(k\pi) = (-1)^k$. Applying a change of variable $\tau$ to $kT + \tau$, we get

$$g_1(2kT) = -\int_{-\infty}^{\infty} h(kT + \tau)h(kT - \tau) \sin \left( \frac{\pi}{T}\tau \right) d\tau = 0.$$ 

where the second identity follows since the expression under the integral is an odd function of $\tau$. 
For an odd values of \( n = 2k + 1 \), one finds that
\[
\cos \left( \frac{\pi}{2T} (2k + 1)T \right) = \cos(k\pi + \pi/2) = 0
\]
and, thus,
\[
g_1((2k + 1)T) = (-1)^{k+1} \int_{-\infty}^{\infty} h(\tau)h((2k + 1)T - \tau) \cos \left( \frac{\pi}{T} \tau \right) d\tau.
\]
where we have noted that \( \sin(k\pi + \pi/2) = (-1)^k \). Applying a change of variable \( \tau \) to \( \frac{(2k+1)T}{2} + \tau \), we get
\[
g_1\left( \frac{(2k+1)T}{2} \right) = \int_{-\infty}^{\infty} h\left( \frac{(2k+1)T}{2} + \tau \right)h\left( \frac{(2k+1)T}{2} - \tau \right) \sin \left( \frac{\pi}{T} \tau \right) d\tau = 0.
\]
where the second identity follows since the expression under the integral is an odd function of \( \tau \).
Filter Bank Multicarrier (FBMC) Methods: CMT (Details)

For an odd values of \( n = 2k + 1 \), one finds that \( \cos \left( \frac{\pi}{2T} (2k + 1)T \right) = \cos(k\pi + \pi/2) = 0 \) and, thus,

\[
g_1((2k + 1)T) = (-1)^{k+1} \int_{-\infty}^{\infty} h(\tau)h((2k + 1)T - \tau) \cos \left( \frac{\pi}{T} \tau \right) d\tau.
\]

where we have noted that \( \sin(k\pi + \pi/2) = (-1)^k \). Applying a change of variable \( \tau \) to \( \frac{(2k+1)T}{2} + \tau \), we get

\[
g_1\left(\frac{(2k+1)T}{2}\right) = \int_{-\infty}^{\infty} h\left(\frac{(2k+1)T}{2} + \tau\right)h\left(\frac{(2k+1)T}{2} - \tau\right) \sin \left( \frac{\pi}{T} \tau \right) d\tau = 0.
\]

where the second identity follows since the expression under the integral is an odd function of \( \tau \).

Combining the above results, one finds that

\[
g_1(nT) = 0
\]

for all integer values of \( n \), which is the condition we required for ICI free transmission.
Filter Bank Multicarrier (FBMC) Methods: CMT (Details)

In CMT data symbols are PAM and modulated into a set of vestigial sideband subcarrier channels.
Filter Bank Multicarrier (FBMC) Methods: CMT (Details)

Equalization:
Filter Bank Multicarrier (FBMC) Methods: CMT (Details)

Blind equalization

Before equalization

After equalization

Filtered Multitone (Details)
Filter Bank Multicarrier (FBMC) Methods: FMT (Synthesis/TX)

Transmitter:
Filter Bank Multicarrier (FBMC) Methods: FMT (Analysis/RX)

Receiver:
Prototype Filter Design
Prototype Filter Design: Design for time-invariant channels

- **Design Strategy**: Design a filter with a minimum bandwidth that satisfies the square-root Nyquist criterion.

An effective design procedure is presented in the next few slides.
Prototype Filter Design: Design for time-invariant channels

Square-root Nyquist (M) filter design:

\[ h = [h[0] \ h[1] \ \cdots \ h[N]]^T \]

\[ H(z) = h^T e(z) \]

\[ G(z) = \left( h^T e(z) \right) \left( h^T e(z^{-1}) \right) \]
\[ = h^T e(z) e^T (z^{-1}) h \]
\[ = h^T R(z) h \]

\( G(z) \) must be a Nyquist (M) filter.
Prototype Filter Design: Design for time-invariant channels

Square-root Nyquist (M) filter design:

\[ R(z) = e(z)e^T(z^{-1}) \]

\[ = \begin{pmatrix} 1 \\ z^{-1} \\ \vdots \\ z^{-N} \end{pmatrix} \begin{pmatrix} 1 & z & \cdots & z^N \end{pmatrix} \]

\[ = \sum_{n=-N}^{N} z^{-n}S_n \]

\[ [S_n]_{k,l} = \begin{cases} 1, & k - l = n \\ 0, & \text{otherwise} \end{cases} \]

Hence,

\[ G(z) = \sum_{n=-N}^{N} \left( h^T S_n h \right) z^{-n} \]

And for \( h \) to be square-root Nyquist (M) filter:

\[ h^T S_n h = \begin{cases} 1, & n = 0 \\ 0, & n = mM, m \neq 0 \end{cases} \]
Prototype Filter Design: Design for time-invariant channels

Square-root Nyquist (M) filter design:

It is also to minimize the stopband gain

\[ \xi_s = \int_{f_2}^{1-f_2} |H(e^{j2\pi f})|^2 df = h^T h - \int_{-f_2}^{f_2} |H(e^{i2\pi f})|^2 df \]

\[ \int_{-f_2}^{f_2} |H(e^{j2\pi f})|^2 df = \int_{-f_2}^{f_2} h^T e(e^{j2\pi f}) e^T(e^{-j2\pi f}) h df \]

\[ = h^T \left( \int_{-f_2}^{f_2} e(e^{j2\pi f}) e^T(e^{-j2\pi f}) df \right) h. \]

Hence,

\[ \xi_s = h^T \Phi h \quad \text{where} \quad \Phi = I - \int_{-f_2}^{f_2} e(e^{j2\pi f}) e^T(e^{-j2\pi f}) df \]

\[ \phi_{kl} = \begin{cases} 1 - 2f_2, & k = l \\ -2f_2 \text{sinc}(2f_2(k - l)), & k \neq l. \end{cases} \]
Prototype Filter Design: Design for time-invariant channels

Square-root Nyquist (M) filter design:

$h$ may be optimized by minimizing

$$\xi = \xi_s + \gamma \sum_{n \in \text{C.P.}} (h^T S_n h - d_n)^2$$

where the terms under summation are the Nyquist constraints, C.P. denotes constraint points and $\gamma$ is a weight factor.

Also,

$$d_n = \begin{cases} 
1, & n = 0 \\
0, & n = mM, \ m \neq 0 
\end{cases}$$

The minimization can be done through an iterative method.

Prototype Filter Design: Design for doubly dispersive channels

Channel spreading function: $S(\tau, \nu)$
Prototype Filter Design: Design for doubly dispersive channels

Channel spreading function: $S(\tau, \nu)$

Design $h(t)$ to match the time and frequency spread of channel.
Prototype Filter Design: Design for doubly dispersive channels

A few definitions:

Time spread

\[ \sigma_t = \sqrt{\int_{-\infty}^{\infty} t^2 |h(t)|^2 dt} \]

Frequency spread

\[ \sigma_f = \sqrt{\int_{-\infty}^{\infty} f^2 |H(f)|^2 df} \]
Prototype Filter Design: Design for doubly dispersive channels

It is reasonable to have:

\[
\frac{\sigma_t}{\Delta \tau} = \frac{\sigma_f}{\Delta \nu}
\]

Also, treating time and frequency the same way, one may choose \( h(t) \) so that

\[
H(f) = h(\ell f), \quad \text{for a constant scaling factor } \ell
\]

Then, one may find that

\[
\frac{T}{\Delta \tau} = \frac{F}{\Delta \nu}
\]

or

\[
\frac{T}{F} = \frac{\Delta \tau}{\Delta \nu} = \frac{\sigma_t}{\sigma_f} = \ell
\]
Prototype Filter Design: Design for doubly dispersive channels

The Gaussian pulse:

\[ g(t) = e^{-\pi t^2} \]

The first property:

\[ G(f) = g(f) \]

The second property: the Heisenberg-Gabor uncertainty principle states that for an arbitrary pulse \( h(t) \)

\[ \sigma_t \sigma_f \geq \frac{1}{4\pi} \]

and equality holds when

\[ h(t) = g(t) \]

However, \( g(t) \) does not possess the property necessary for ISI/ICI Cancellation.
Prototype Filter Design: Design for doubly dispersive channels

Generalized Nyquist criterion:

An FBMC signal

\[ x(t) = \sum_n \sum_{k \in K} s_k[n] h_k(t - nT), \quad h_k(t) = h(t) e^{j2\pi tkF} \]

\[ < h_k(t - mT), h_l(t - nT) > = \delta_{kl} \delta_{mn} \quad (a \ set \ of \ orthogonal \ bases) \]

The ambiguity function:

\[ A_h(\tau, \nu) = \int_{-\infty}^{\infty} h(t + \tau/2)h(t - \tau/2)e^{-j2\pi \nu t} dt \]

\[ < h_k(t - mT), h_l(t - nT) > = \int_{-\infty}^{\infty} h(t - mT)e^{j2\pi kFt} h(t - nT)e^{-j2\pi lFt} dt \]

\[ \propto A_h((n - m)T, (l - k)F) \]

\[ A_h(nT, lF) = \begin{cases} 
1, & n = l = 0 \\
0, & \text{otherwise}.
\end{cases} \]

The generalized Nyquist criterion
Prototype Filter Design: Design for doubly dispersive channels

The time-frequency grid:
Prototype Filter Design: Design for doubly dispersive channels

Isotropic filters: a filter is called isotropic, if

\[ H(f) = h(\ell f), \quad \text{for a constant scaling factor } \ell \]

The first design: isotropic orthogonal transform algorithm (IOTA)

\[ h(t) = \mathcal{F}^{-1} O_T \mathcal{F} O_F g(t) \]

\[ O_a : \quad y(u) = \frac{x(u)}{\sqrt{\frac{1}{a} \sum_{k=-\infty}^{\infty} |x(u - k/a)|^2}} \]
Prototype Filter Design: Design for doubly dispersive channels

Isotropic filters:

The second design:

\[ h(t) = \sum_{k=0}^{L} a_k h_{4k}(t) \]

\[ h_n(t) = \frac{1}{(2\pi)^{n/2}} e^{\pi t^2} \frac{d^n}{dt^n} e^{-2\pi t^2} \quad \text{(Hermite functions)} \]
Prototype Filter Design: Design for doubly dispersive channels

Isotropic filters:

The second design: the algorithm

\[
A_p(\tau, \nu) = \int_{-\infty}^{\infty} \sum_{n=0}^{L} \sum_{l=0}^{L} a_n a_l h_{4n}(t + \frac{\tau}{2}) h_{4l}(t - \frac{\tau}{2}) e^{-j2\pi \nu t} dt
\]

\[
= \sum_{n=0}^{L} \sum_{l=0}^{L} a_n a_l A_{n,l}(\tau, \nu)
\]

\[
= a^T A(\tau, \nu) a
\]

where

\[
A(\tau, \nu) = \begin{bmatrix}
A_{0,0}(\tau, \nu) & A_{0,1}(\tau, \nu) & \cdots & A_{0,L}(\tau, \nu) \\
A_{1,0}(\tau, \nu) & A_{1,1}(\tau, \nu) & \cdots & A_{1,L}(\tau, \nu) \\
\vdots & \vdots & \ddots & \vdots \\
A_{L,0}(\tau, \nu) & A_{L,1}(\tau, \nu) & \cdots & A_{L,L}(\tau, \nu)
\end{bmatrix}
\]

\[
a^T A(nT, lF) a = \begin{cases} 
1, & n = l = 0 \\
0, & \text{otherwise}
\end{cases}
\]

and

\[
A_{n,l}(\tau, \nu) = \int_{-\infty}^{\infty} h_{4n}(t + \frac{\tau}{2}) h_{4l}(t - \frac{\tau}{2}) e^{-j2\pi \nu t} dt
\]
Prototype Filter Design: Design for doubly dispersive channels

Isotropic filters:

The second design: the algorithm

\[ A_p(\tau, \nu) = \int_{-\infty}^{\infty} \sum_{n=0}^{L} \sum_{l=0}^{L} a_n a_l h_4n(t + \frac{\tau}{2}) h_4l(t - \frac{\tau}{2}) e^{-j2\pi \nu t} dt \]

\[ = \sum_{n=0}^{L} \sum_{l=0}^{L} a_n a_l A_{n,l}(\tau, \nu) \]

\[ = a^T A(\tau, \nu) a \]

where

\[ A(\tau, \nu) = \begin{bmatrix} A_{0,0}(\tau, \nu) & A_{0,1}(\tau, \nu) & \cdots & A_{0,L}(\tau, \nu) \\ A_{1,0}(\tau, \nu) & A_{1,1}(\tau, \nu) & \cdots & A_{1,L}(\tau, \nu) \\ \vdots & \vdots & \ddots & \vdots \\ A_{L,0}(\tau, \nu) & A_{L,1}(\tau, \nu) & \cdots & A_{L,L}(\tau, \nu) \end{bmatrix} \]

To design \( h(t) \), we should find the coefficients \( a_n \) through solving the set of simultaneous equations

\[ a^T A(nT, lF) a = \begin{cases} 1, & n = l = 0 \\ 0, & \text{otherwise} \end{cases} \]

and

\[ A_{n,l}(\tau, \nu) = \int_{-\infty}^{\infty} h_4n(t + \frac{\tau}{2}) h_4l(t - \frac{\tau}{2}) e^{-j2\pi \nu t} dt \]
Prototype Filter Design: Design for doubly dispersive channels

Isotropic filters:

The second design: the algorithm

\[ A_p(\tau, \nu) = \int_{-\infty}^{\infty} \sum_{n=0}^{L} \sum_{l=0}^{L} a_n a_l h_{4n}(t + \frac{\tau}{2}) h_{4l}(t - \frac{\tau}{2}) e^{-j2\pi\nu t} dt \]

\[ = \sum_{n=0}^{L} \sum_{l=0}^{L} a_n a_l A_{n,l}(\tau, \nu) \]

\[ = a^T A(\tau, \nu) a \]

where

\[ A(\tau, \nu) = \begin{bmatrix} A_{0,0}(\tau, \nu) & A_{0,1}(\tau, \nu) & \cdots & A_{0,L}(\tau, \nu) \\ A_{1,0}(\tau, \nu) & A_{1,1}(\tau, \nu) & \cdots & A_{1,L}(\tau, \nu) \\ \vdots & \vdots & \ddots & \vdots \\ A_{L,0}(\tau, \nu) & A_{L,1}(\tau, \nu) & \cdots & A_{L,L}(\tau, \nu) \end{bmatrix} \]

To design \( h(t) \), we should find the coefficients \( a_n \) through solving the set of simultaneous equations

\[ a^T A(nT, lF) a = \begin{cases} 1, & n = l = 0 \\ 0, & \text{otherwise} \end{cases} \]

Procedure will be similar to that of square-root Nyquist filter.

\[ A_{n,l}(\tau, \nu) = \int_{-\infty}^{\infty} h_{4n}(t + \frac{\tau}{2}) h_{4l}(t - \frac{\tau}{2}) e^{-j2\pi\nu t} dt \]
Prototype Filter Design: Design for doubly dispersive channels

The time-frequency grid:

- Constraints are applied at the solid circles encircled by another circle.
- Constraints will be imposed at solid circles because of isotropic property.
- Approximate constraints are imposed at other points because of the exponential decay of the Hermite functions.
Prototype Filter Design: Design for doubly dispersive channels

Numerical results (without channel effect):

Design for perfect zeros at the grid points  Design for approx zeros at the grid points
Prototype Filter Design: Design for doubly dispersive channels

Numerical results (with channel effect):

Design for perfect zeros at the grid points

Design for approx zeros at the grid points
Implementation of Multicarrier Filter Banks
Implementation of MCFB: Basic structure (synthesis/TX)

(continuous-time)

\[ s_k(t) = \sum_{n=-\infty}^{\infty} s_k[n] \delta(t - nT), \quad \text{for } k = 0, 1, \ldots, N - 1 \]
Implementation of MCFB: Basic structure (synthesis/TX)

(discrete-time)
Implementation of MCFB: Basic structure (synthesis/TX)

Development of the polyphase structure: consider the kth branch of the synthesis filter bank of the previous slide
Implementation of MCFB: Basic structure (synthesis/TX)

Development of the polyphase structure: consider the kth branch of the synthesis filter bank of the previous slide

\[ x_k[n] = \left( \sum_{m=-\infty}^{\infty} s_k[m]h[n - mL] \right) e^{j \frac{2k\pi}{L} n}. \]

Noting that \( e^{j \frac{2k\pi}{L} n} = e^{j \frac{2k\pi}{L} (n-mL)} \), we obtain \( x_k[n] = \sum_{m=-\infty}^{\infty} s_k[m]h_k[n-mL] \).

where \( h_k[n] = h[n]e^{j \frac{2k\pi}{L} n} \). We note that \( h_k[n] \) is a modulated version of \( h[n] \).
Implementation of MCFB: Basic structure (synthesis/TX)

Development of the polyphase structure: consider the kth branch of the synthesis filter bank of the previous slide

\[ x_k[n] = \left( \sum_{m=-\infty}^{\infty} s_k[m] h[n - mL] \right) e^{j \frac{2k\pi}{L} n} \]

Noting that \( e^{j \frac{2k\pi}{L} n} = e^{j \frac{2k\pi}{L} (n - mL)} \), we obtain \( x_k[n] = \sum_{m=-\infty}^{\infty} s_k[m] h_k[n - mL] \), where \( h_k[n] = h[n] e^{j \frac{2k\pi}{L} n} \). We note that \( h_k[n] \) is a modulated version of \( h[n] \).
Implementation of MCFB: Basic structure (synthesis/TX)

Development of the polyphase structure: consider the kth branch of the synthesis filter bank of the previous slide.

\[ x_k[n] = \left( \sum_{m=-\infty}^{\infty} s_k[m]h[n-mL] \right) e^{j\frac{2k\pi}{L}n}. \]

Noting that \( e^{j\frac{2k\pi}{L}n} = e^{j\frac{2k\pi}{L}(n-mL)} \), we obtain \( x_k[n] = \sum_{m=-\infty}^{\infty} s_k[m]h_k[n-mL] \).

where \( h_k[n] = h[n]e^{j\frac{2k\pi}{L}n} \). We note that \( h_k[n] \) is a modulated version of \( h[n] \).

\[ X_k(z) = S_k(z^L)H_k(z) \]
Derivation of the synthesis polyphase filter bank

We note that \( X(z) = \sum_{k=0}^{N-1} X_k(z) \). Also, \( H_k(z) = \sum_n h[n] z^{-n} W_L^{-kn} \), where \( W_L = e^{-j2\pi/L} \).

From these, we obtain

\[
X(z) = \sum_{k=0}^{N-1} \sum_n S_k (z^L) h[n] z^{-n} W_L^{-kn}.
\]

Next, we introduce the change of variable \( n = mL + l \), to obtain

\[
X(z) = \sum_{k=0}^{N-1} \sum_{l=0}^{L-1} S_k (z^L) W_L^{-kl} E_l(z^L) z^{-l}
\]

where \( E_l(z) \) is the \( l \)th polyphase component of \( H(z) \), defined as

\[
E_l(z) = \cdots + h[l - L]z + h[l] + h[l + L]z^{-1} + \cdots
\]

From these, we obtain the results presented on the next slide.
Implementation of MCFB: Basic structure (synthesis/TX)

\[ X(z) = \begin{bmatrix} S_0(z^L) & S_1(z^L) & \cdots & S_{N-1}(z^L) & 0 & \cdots & 0 \end{bmatrix} \mathcal{F}^* \]

\[ \times \begin{bmatrix} E_0(z^L) & 0 & \cdots & 0 \\ 0 & E_1(z^L) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & E_{L-1}(z^L) \end{bmatrix} \begin{bmatrix} 1 \\ z^{-1} \\ \vdots \\ z^{-(L-1)} \end{bmatrix} . \]

where \( \mathcal{F} \) is the \( L \times L \) Fourier transform matrix defined as

\[ \mathcal{F} = \begin{bmatrix} 1 & 1 & \cdots & 1 \\ 1 & W_L & \cdots & W_L^{L-1} \\ \vdots & \vdots & \ddots & \vdots \\ 1 & W_L^{L-1} & \cdots & W_L^{(L-1)(L-1)} \end{bmatrix} . \]
Implementation of MCFB: Basic structure (synthesis/TX)

Using the equations on the previous slide, we obtain the following structure...
Implementation of MCFB: Basic structure (synthesis/TX)

\[
X(z) = \begin{bmatrix}
S_0(z^L) & S_1(z^L) & \cdots & S_{N-1}(z^L) & 0 & \cdots & 0
\end{bmatrix} \mathcal{F}^* \times \begin{bmatrix}
E_0(z^L) & 0 & \cdots & 0 \\
0 & E_1(z^L) & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & E_{L-1}(z^L)
\end{bmatrix} \begin{bmatrix}
1 \\
0 \\
\vdots \\
0
\end{bmatrix}
\]

where \( \mathcal{F} \) is the \( L \times L \) Fourier transform matrix defined as

\[
\mathcal{F} = \begin{bmatrix}
1 & 1 & \cdots & 1 \\
1 & W_L & \cdots & W_L^{L-1} \\
\vdots & \vdots & \ddots & \vdots \\
1 & W_L^{L-1} & \cdots & W_L^{(L-1)(L-1)}
\end{bmatrix}.
\]
Implementation of MCFB: Basic structure (synthesis/TX)

Moving the expander to the output side:
Implementation of MCFB: Basic structure (analysis/RX)

Note that $X_k(z) = S_k(z^L)H_k(z)$ and

$$X(z) = \sum_{k=0}^{N-1} S_k(z^L)H_k(z)$$

$$= \begin{bmatrix} S_0(z^L) & S_1(z^L) & \cdots & S_{N-1}(z^L) & 0 & \cdots & 0 \end{bmatrix} \begin{bmatrix} H_0(z) \\ H_1(z) \\ \vdots \\ H_{L-1}(z) \end{bmatrix}.$$ 

Hence,

$$\begin{bmatrix} H_0(z) \\ H_1(z) \\ \vdots \\ H_{L-1}(z) \end{bmatrix} = \mathcal{F}^* \times \begin{bmatrix} E_0(z^L) & 0 & \cdots & 0 \\ 0 & E_1(z^L) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & E_{L-1}(z^L) \end{bmatrix} \begin{bmatrix} 1 \\ z^{-1} \\ \vdots \\ z^{-(L-1)} \end{bmatrix}.$$
Implementation of MCFB: Basic structure (analysis/RX)

At the receiver:

\[
\begin{bmatrix}
S_0(z^L) \\
S_1(z^L) \\
\vdots \\
S_{N-1}(z^L) \\
\ast \\
\vdots \\
\ast
\end{bmatrix}
= 
\begin{bmatrix}
H_0(z) \\
H_1(z) \\
\vdots \\
H_{L-1}(z)
\end{bmatrix} 
X(z)
\]

\[
= \mathcal{F}^* \times 
\begin{bmatrix}
E_0(z^L) & 0 & \cdots & 0 \\
0 & E_1(z^L) & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & E_{L-1}(z^L)
\end{bmatrix}
\times
\begin{bmatrix}
1 \\
\vdots \\
z^{-(L-1)}
\end{bmatrix} 
X(z).
\]
Implementation of MCFB: Basic structure (analysis/RX)

Using the equation on the previous slide, the following structure is obtained:
Implementation of MCFB: Basic structure (analysis/RX)

\[
\begin{align*}
\begin{bmatrix}
S_0(z^L) & S_1(z^L) & \cdots & S_{N-1}(z^L)
\end{bmatrix} &= \begin{bmatrix}
H_0(z) & H_1(z) & \cdots & H_{L-1}(z)
\end{bmatrix} X(z) \\
&= \mathcal{F}^* \times \begin{bmatrix}
E_0(z^L) & 0 & \cdots & 0 \\
0 & E_1(z^L) & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & E_{L-1}(z^L)
\end{bmatrix} \times \begin{bmatrix}
1 \\
z^{-1} \\
\vdots \\
z^{-(L-1)}
\end{bmatrix} X(z),
\end{align*}
\]
Staggered Modulated Multitone (SMT)
Implementation of MCFB: SMT (synthesis/transmitter)

Recall:

And note that, ignoring modulation to RF band, this can be rearranged as in the next slide.
Implementation of MCFB: SMT (synthesis/transmitter)
Implementation of MCFB: SMT (synthesis/transmitter)

or

SFB: Synthesis filter bank
Implementation of MCFB: SMT (analysis/receiver)
Implementation of MCFB: SMT (analysis/receiver)
Cosine Modulated Multitone (CMT)
Implementation of MCFB: CMT (synthesis/transmitter)
Implementation of MCFB: CMT (synthesis/transmitter)

\[ s_k(t) = \sum_{n=-\infty}^{\infty} s_k[n] \delta(t - nT) \]

\[ v_k(t) = \sum_{n=-\infty}^{\infty} s_k[n] h(t - nT) e^{j \frac{\pi}{2T} (t - nT)} \]

\[ = \left( \sum_{n=-\infty}^{\infty} (-j)^n s_k[n] h(t - nT) \right) e^{j \frac{\pi}{2T} t} \]
Implementation of MCFB: CMT (synthesis/transmitter)

Using the results of the previous slide, the following structure is obtained.

Notes:
• The phase shifts of $\pi/2$ in the modulators have been extracted and added to the inputs.
• The subcarrier spacing is $\pi/T$ (not $2\pi/T$).
Implementation of MCFB: CMT (synthesis/transmitter)

\[
x_k[n] = \left( \sum_{m=-\infty}^{\infty} j^k (-j)^m s_k[m] h[n - mL] \right) e^{j \frac{k \pi}{L} n}
\]

This can be rearranged as

\[
x_k[n] = \sum_{m=-\infty}^{\infty} j^k (-j)^m (-1)^{mk} s_k[m] h_k[n - mL]
\]

where

\[
h_k[n] = h[n] e^{j \frac{k \pi}{L} n}
\]

is a modulated version of \( h[n] \).
Implementation of MCFB: CMT (synthesis/transmitter)

From the results of the previous slide, we obtain

\[
j^k (-j)^n \tilde{s}_k[n] \xrightarrow{\uparrow L} h_k[n] \xrightarrow{} x_k[n]
\]

\[
\tilde{s}_k[n] = \begin{cases} 
  s_k[n], & \text{for } k \text{ even} \\
  (-1)^n s_k[n], & \text{for } k \text{ odd}
\end{cases}
\]

In terms of the z-domain variable, one finds that

\[
X_k(z) = j^k \tilde{S}_k \left(jz^L\right) H_k(z)
\]

where it has been noted the z-transform of \((-j)^n \tilde{s}_k[n]\) is equal to \(\tilde{S}_k(-z/j) = \tilde{S}_k(jz)\) and after the expander \(z\) is replaced by \(z^L\). Also, \(j^k\) is a constant factor.
Implementation of MCFB: CMT (synthesis/transmitter)

The relationship \( x[n] = x'[n]e^{j\frac{2\pi}{2L}n} = x'[n]W_{2L}^{-\frac{1}{2}n} \) implies

\[
X(z) = X'(zW_{2L}^{\frac{1}{2}}).
\]

We also note that

\[
X'(z) = \sum_{k=0}^{N-1} X_k(z).
\]

Moreover,

\[
H_k(z) = \sum_n h[n]z^{-n}W_{2L}^{-kn}
\]

where \( W_{2L} = e^{-j\frac{2\pi}{2L}} = e^{-j\pi/L} \). From these, we obtain

\[
X'(z) = \sum_{k=0}^{N-1} \sum_n j^k S_k (jz^L) h[n]z^{-n}W_{2L}^{-kn}.
\]

Introducing the change of variable \( n = 2mL + l \), we get

\[
X'(z) = \sum_{k=0}^{N-1} \sum_{l=0}^{2L-1} j^k S_k (jz^L) W_{2L}^{kL} E_l(z^{2L})z^{-l}
\]

where \( E_l(z) \) is the \( l \)-th polyphase component of \( H(z) \), defined as

\[
E_l(z) = \cdots + h[l - 2L]z + h[l] + h[l + 2L]z^{-1} + \cdots
\]

Note that here the decimation factor is \( 2L \).
Implementation of MCFB: CMT (synthesis/transmitter)

\[
X'(z) = \begin{bmatrix}
\tilde{S}_0(jz^L) & j\tilde{S}_1(jz^L) & \cdots & j^{N-1}\tilde{S}_{N-1}(jz^L) & 0 & \cdots & 0
\end{bmatrix} \mathcal{F}^* \\
\begin{bmatrix}
E_0(z^{2L}) & 0 & \cdots & 0 \\
0 & E_1(z^{2L}) & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & E_{2L-1}(z^{2L})
\end{bmatrix} \\
\begin{bmatrix}
1 \\
z^{-1} \\
\vdots \\
z^{-(2L-1)}
\end{bmatrix}
\]

where \( \mathcal{F} \) is the \( 2L \times 2L \) Fourier transform matrix defined as

\[
\mathcal{F} = \begin{bmatrix}
1 & 1 & \cdots & 1 \\
1 & W_{2L} & \cdots & W_{2L}^{2L-1} \\
\vdots & \vdots & \ddots & \vdots \\
1 & W_{2L}^{2L-1} & \cdots & W_{2L}^{(2L-1)(2L-1)}
\end{bmatrix}.
\]
Implementation of MCFB: CMT (synthesis/transmitter)

\[
X(z) = X'(zW_{2L}^{1/2})
\]

\[
= \left[ \tilde{S}_0(z^L) j \tilde{S}_1(z^L) \cdots j^{N-1} \tilde{S}_{N-1}(z^L) 0 \cdots 0 \right] F^* 
\]

\[
\times \begin{bmatrix}
E_0(-z^{2L}) & 0 & \cdots & 0 \\
0 & E_1(-z^{2L}) & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & E_{2L-1}(-z^{2L})
\end{bmatrix}
\]

\[
\times \begin{bmatrix}
1 \\
W_{2L}^{-1/2} z^{-1} \\
\vdots \\
W_{2L}^{-2L-1/2} z^{-(2L-1)}
\end{bmatrix}
\]

where we have noted that \( j \left( zW_{2L}^{1/2} \right)^L = z^L \).
Implementation of MCFB: CMT (synthesis/transmitter)

The above results leads to the synthesis/transmitter structure:
Implementation of MCFB: CMT (synthesis/transmitter)

\[
X(z) = \begin{bmatrix}
\tilde{S}_0(z^L) & j \tilde{S}_1(z^L) & \cdots & j^{N-1} \tilde{S}_{N-1}(z^L) & 0 & \cdots & 0
\end{bmatrix} Z^* \begin{bmatrix}
E_0(-z^{2L}) & 0 & \cdots & 0 \\
0 & E_1(-z^{2L}) & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & E_{2L-1}(-z^{2L})
\end{bmatrix} \begin{bmatrix}
\frac{1}{W_{2L}^2 z^{-1}} \\
\cdots \\
W_{2L}^{-\frac{1}{2}} z^{-(2L-1)}
\end{bmatrix}
\]
Implementation of MCFB: CMT (analysis/receiver)

The polyphase structure of a CMT receiver:
Implementation of MCFB: CMT (analysis/receiver)

The multiplications by 1, $-j$, $\cdots$, $(-j)^{N-1}$, before taking the real parts, is to compensate for the coefficients 1, $j$, $\cdots$, $j^{N-1}$ at the transmitter.

$$H(z) = \mathcal{F}^* \times \begin{bmatrix}
E_0(-z^{2L}) & 0 & \cdots & 0 \\
0 & E_1(-z^{2L}) & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & E_{2L-1}(-z^{2L})
\end{bmatrix} \times \begin{bmatrix}
1 \\
W_{2L}^{-\frac{1}{2}} z^{-1} \\
\vdots \\
W_{2L}^{-\frac{2L-1}{2}} z^{-(2L-1)}
\end{bmatrix}$$
Implementation of MCFB: FMT (Synthesis/transmitter)

Filtered Multitone (FMT)
Implementation of MCFB: FMT (Synthesis/transmitter)

Transmitter:
Implementation of MCFB: FMT (Analysis/receiver)

Receiver:
Implementation of MCFB: FMT (Synthesis/transmitter)

Case of $\alpha = 1$:
Implementation of MCFB: FMT (Synthesis/transmitter)

Case of $\alpha = 1$:

Computational saving can be made, using the fact that alternate inputs are zero.
Implementation of MCFB: FMT (Analysis/receiver)

Case of $\alpha = 1$:
Implementation of MCFB: FMT (Analysis/receiver)

Case of $\alpha = 1$:

Computational saving can be made, noting that only alternate outputs should be calculated.
Implementation of MCFB: FMT (Synthesis/transmitter)

Case of $\alpha = 0.5$: 

\[ s_0[n] \rightarrow s_1[n] \rightarrow s_2[n] \rightarrow s_3[n] \]

\[ f \]

\[ 0, \frac{3}{2T}, \frac{3}{T}, \frac{3}{2T} + \frac{3}{T} \]

\[ s_0[n], 0, 0, 0 \]

\[ s_1[n], 0, 0, 0 \]

\[ s_2[n], 0, 0, 0 \]

\[ s_3[n], 0, 0, 0 \]

\[ x[n] = e^{j2\pi \frac{3}{2T} n} \]
Implementation of MCFB: FMT (Analysis/Receiver)

Case of $\alpha = 0.5$:
Implementation of MCFB: FMT

- The implementations that were introduced in the past few slides, although simple to understand, are computationally inefficient.

- In the next few slides, we present the polyphase structures that offer the least complexity.

- These polyphase structures are different from those presented so far in the sense that they use polyphase components that vary with time. Thus, may be classified as time-varying polyphase structures.
Implementation of MCFB: FMT

Spectrum of an FMT signal. In (a) the frequency axis is in the units of Hertz, and in (b) it is normalized.
Implementation of MCFB: FMT (Synthesis/transmitter)
Implementation of MCFB: FMT (Synthesis/transmitter)

A conventional (time-invariant) polyphase structure cannot be adopted here because $K \neq N$. 
Implementation of MCFB: FMT (Synthesis/transmitter)

Following the figure on the previous slide, we obtain

\[ x[n] = \sum_{k=0}^{N-1} \left( \sum_{m} s_k[m] h[n - mK] \right) e^{j \frac{2\pi km}{N}} \]

\[ = \sum_{m} \left( \sum_{k=0}^{N-1} s_k[m] e^{j \frac{2\pi km}{N}} \right) h[n - mK]. \]

To proceed, we write

\[ n = \gamma N + p \]

where \( \gamma \) is the integer part of \( n/N \) and \( p = 0, 1, \cdots, N - 1 \) is its remainder, and also

\[ n = \eta K + q \]

where \( \eta \) is the integer part of \( n/K \) and \( q = 0, 1, \cdots, K - 1 \) is its remainder.

Combining the above equations, we obtain

\[ x[\eta K + q] = \sum_{m} S_p[m] h[(\eta - m)K + q], \quad q = 0, 1, \cdots, K - 1 \]

where

\[ S_p[m] = \sum_{k=0}^{N-1} s_k[m] e^{j \frac{2\pi kp}{N}}, \quad p = 0, 1, \cdots, N - 1. \]
Implementation of MCFB: FMT (Synthesis/transmitter)

- Perform an IDFT on the input symbols \( s_0[n] \), \( s_1[n] \), \( \cdots \), \( s_{N-1}[n] \).
- Pass the output samples from the IDFT through the polyphase components.
- The commutator takes \( K \) steps after arrival of each set of input symbols.
- \( D_0^{(n)}(z) \) through \( D_{N-1}^{(n)}(z) \) are a set of time-varying filters that are picked up from the set of \( K \) polyphase components \( E_0(z) \) through \( E_{K-1}(z) \) of the prototype filter \( H(z) = \sum_n h[n]z^{-n} \).
- For a given time \( n \), \( x[n] \) is taken from the output of \( D_p^{(n)}(z) \) and \( D_p^{(n)}(z) = E_q(z) \), where \( p \) and \( q \) are the integers defined on the previous slide.
A conventional (time-invariant) polyphase structure cannot be adopted here because $K \neq N$. 
Implementation of MCFB: FMT (Analysis/Receiver)

Here,

\[ y_k[n] = \left( y[n]e^{-j \frac{2\pi kn}{N}} \right) \ast h[n] \]
\[ = \sum_m y[m]e^{-j \frac{2\pi km}{N}} h[n - m]. \]

Substituting \( m = \gamma N + p \), where \( \gamma \) is the integer part of \( m/N \) and \( p = 0, 1, \cdots, N - 1 \) is the remainder part of \( m/N \), we get

\[ y_k[n] = \sum_{\gamma} \sum_{p=0}^{N-1} y[\gamma N + p]e^{-j \frac{2\pi kp}{N}} h[n - \gamma N - p] \]
\[ = \sum_{p=0}^{N-1} \left( \sum_{\gamma} y[\gamma N + p]h[n - \gamma N - p] \right) e^{-j \frac{2\pi kp}{N}}. \]

Also, substituting \( n - p = \kappa N + q \), where \( \kappa \) is the integer part of \((n - p)/N\) and \( q = 0, 1, \cdots, N - 1 \) is the remainder of \( n - p \) divide by \( N \), we obtain

\[ y_k[n] = \sum_{p=0}^{N-1} \left( \sum_{\gamma} y[\gamma N + p]h[(\kappa - \gamma)N + q] \right) e^{-j \frac{2\pi kp}{N}}. \]
Implementation of MCFB: FMT (Analysis/Receiver)

- The commutator takes the successive samples of the received signal $y[n]$ and distributes them across the inputs of the time-varying filters $D_0^{(n)}(z)$ through $D_{N-1}^{(n)}(z)$.

- The filters $D_0^{(n)}(z)$ through $D_{N-1}^{(n)}(z)$ are picked up from the set of $N$ polyphase components $E_0(z)$ through $E_{N-1}(z)$ of the prototype filter $H(z) = \sum_n h[n] z^{-n}$.

- For a given time $n$, $D_p^{(n)}(z) = E_q(z)$, where $q$ is the remainder of $n - p$ divide by $N$. 
Conclusions

- We presented the general concept of multicarrier communication and channel sensing for CR networks.
  - Conventional OFDM needs many costly fixes before it can become applicable in CR networks

- Three methods of multicarrier communication (FMT, SMT, and CMT) were introduced and studied in detail.
  - These methods offer a number of advantages over OFDM: higher bandwidth efficiency, reduced intercarrier interference.

- Multitaper method (MTM), a near optimum spectral estimation method, was introduced and evaluated.

- We proposed the use of filter banks as a spectral estimation tool.
  - We compared the proposed filter bank spectral estimator with MTM and found that they behave very similarly
Other Relevant Works

In addition to the methods discussed in this tutorial, there is a body of the works that take advantage of the cyclostationarity of digital communication signals to detect such signals when they at a level below the channel noise. Some relevant works are:

AbstractPlus | Full Text: PDF(744 KB) IEEE CNF


Ghozzi, Mohamed; Marx, Francois; Dohler, Mischa; Palicot, Jacques, “Cyclostationarity-Based Test for Detection of Vacant Frequency Bands,” 1st International Conference on Cognitive Radio Oriented Wireless Networks and Communications, 8-10 June 2006, pp. 1 - 5.


The Wireless Communication Lab
THE UNIVERSITY OF UTAH
Thank You!
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### References

<table>
<thead>
<tr>
<th>Reference Code</th>
<th>Author(s)</th>
<th>Title Details</th>
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References


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http://grouper.ieee.org/802.22