Analysis of an Eigenstructure Technique for DSSS Synchronization *

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Abstract

A new technique for synchronization of direct sequence spread spectrum (DSSS) signals is presented. This technique exploits the eigenstructure of a frequency-channelized DSSS signal to directly estimate the underlying spreading code. The synchronous estimate of the spreading code steadily improves with the increase in the collected data. In this paper, the algorithm used for estimating the spreading code is derived. It is shown that under infinite time-average assumptions, a perfect code estimate can be obtained when the signal is received in arbitrary levels of white background noise. The only requirements are that the spreading code must truly multiply the message signal, the code must have a constant modulus, and the message and code repetition rates must be practically incommensurate. It is shown that the technique is insensitive to frequency offsets on the received signal that are integer multiples of the code repetition rate. The technique is also shown to exploit multipath.

1 Introduction

Code synchronization constitutes an important aspect of a direct sequence spread spectrum (DSSS) receiver. It is a process by which the receiver synchronizes to the *pseudonoise* (PN) spreading code of the received signal. The first stage of synchronization, termed *PN acquisition*, involves a coarse estimate of the PN code within a fraction of a chip.

Conventional acquisition techniques [1, 2, 3] exploit the knowledge of the internal structure of the spreading code to achieve synchronization. They exhibit good acquisition performance in low noise environments but are not suitable for environments with high levels of noise and interference. Factors such as Doppler shift and multipath complicate the problem further.

DSSS packet radio and military systems often require frequent, fast and robust synchronization. *Blind* estimation of the spreading code without the *a priori* knowledge of its structure, timing or the Doppler-shift is useful in achieving these objectives. A blind technique directly estimates the delayed and Doppler-shifted code at the receiver.

The *eigenstructure technique* presented here is a blind technique based on the *dominant mode despreader*, first proposed in [4], and estimates the spreading code by exploiting its constant modulus property and the eigenstructure of the frequency-channelized DSSS signal. Under infinite time-average assumptions, the technique provides a perfect estimate of the

*This work was sponsored by Virginia's Center for Innovative Technology and the Research Center for Excellence project from the Federal Highway Administration.

spreading code in the presence of arbitrary levels of temporally-white background noise and for arbitrary codes satisfying the constant modulus property.

This paper presents the development of the algorithm used in the eigenstructure technique. It is shown that the technique is insensitive to frequency offsets in the signal that are greater than half the code repetition rate. Any small residual signal frequency offset can be removed at the despread signal-to-noise ratio (SNR). It is also shown that when the signal is received in a multipath environment, the technique provides a multipath estimate of the code, which may be used to enhance the demodulated signal. Simulation results for different channels are presented in [5, 6].

This paper is organized as follows. In section 2, the algorithm is derived for a signal in a single user environment. In section 3, the algorithm is analyzed for a signal received in a multipath environment. Conclusions are presented in section 4.

2 The Eigenstructure Technique

Linear periodically time varying (LPTV) processing techniques have been developed for the direct extraction of the message signal from the received DSSS signal by employing an algorithm that adapts to the channel to remove the spreading code as part of the demodulation process. The eigenstructure technique for synchronization is based on these techniques.

2.1 Processor Development

The baseband transmitted DSSS signal is modeled as

$$s(t) = c(t)d(t), \tag{1}$$

where $c(t)$ is the spreading code and $d(t)$ is the data (information) signal. The message is recovered at the receiver by

$$\hat{d}(t) = \mathcal{L}_d \circ [c^*(t)x(t)], \tag{2}$$

where $x(t)$ is the received signal and $\mathcal{L}_d$ is a linear lowpass filter operator with frequency response $L_d(f)$ that covers the passband of $d(t)$. The spreading code $c(t)$ is periodic with a repetition rate $f_r = 1/T_r$, such that $c(t+T_r) = c(t)$. Therefore, it can be replaced by its Fourier series expansion:

$$c(t) = \sum_k C(k)e^{j2\pi k f_r t}, \tag{3}$$

where

$$C(k) = \frac{1}{T_r}\int_{-T_r/2}^{T_r/2} c(t)e^{-j2\pi k f_r t}\,dt. \tag{4}$$

Substituting this *Fourier series representation* (FSR) of $c(t)$ into (1) and (2), the spreading and despreading processes may then be expressed as:

$$s(t) = \left[\sum_k C(k)e^{j2\pi k f_r t}\right] d(t)$$

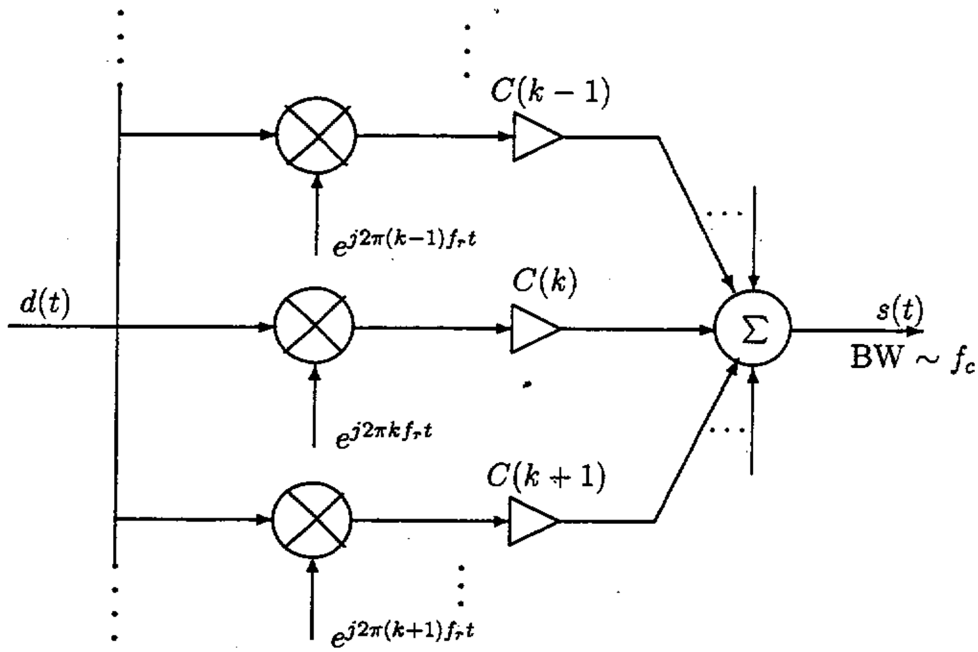

Figure 1: Fourier series representation of the DSSS spreader.

$$= \sum_k C(k)[d(t)e^{j2\pi k f_r t}] \tag{5}$$

$$\hat{d}(t) = \mathcal{L}_d \circ \left\{ \left[\sum_k C(k) e^{j2\pi k f_r t} \right]^* x(t) \right\}$$

$$= \sum_k C^*(k) x_k(t), \tag{6}$$

where

$$x_k(t) = \mathcal{L}_d \circ [x(t)e^{-j2\pi k f_r t}]. \tag{7}$$

The processors implementing these operations (known as the FSRs of the DSSS spreading and despreading operations) are as shown in Figures 1 and 2, respectively. The presence of the frequency-shift operations in the two processors suggest that they are linear periodically time-variant operations. The nonblind time-dependent processor discussed in [7] and [8] has the general form

$$\hat{d}(t) = \sum_k \mathcal{H}_k \circ [x(t)e^{-j2\pi k f_r t}], \tag{8}$$

where $\{\mathcal{H}_k\}$ are a set of LTI filters. The FSR despreader here may be modeled by this processor but with the *constraint*

$$H_k(f) = C^*(k) L_d(f). \tag{9}$$

The adjustment of the weights of the FIR filter bank implementing the LTI filters to minimize the error between the despread signal $\hat{d}(t)$ and the original signal $d(t)$ (as done in [8]) requires the knowledge of the message signal.

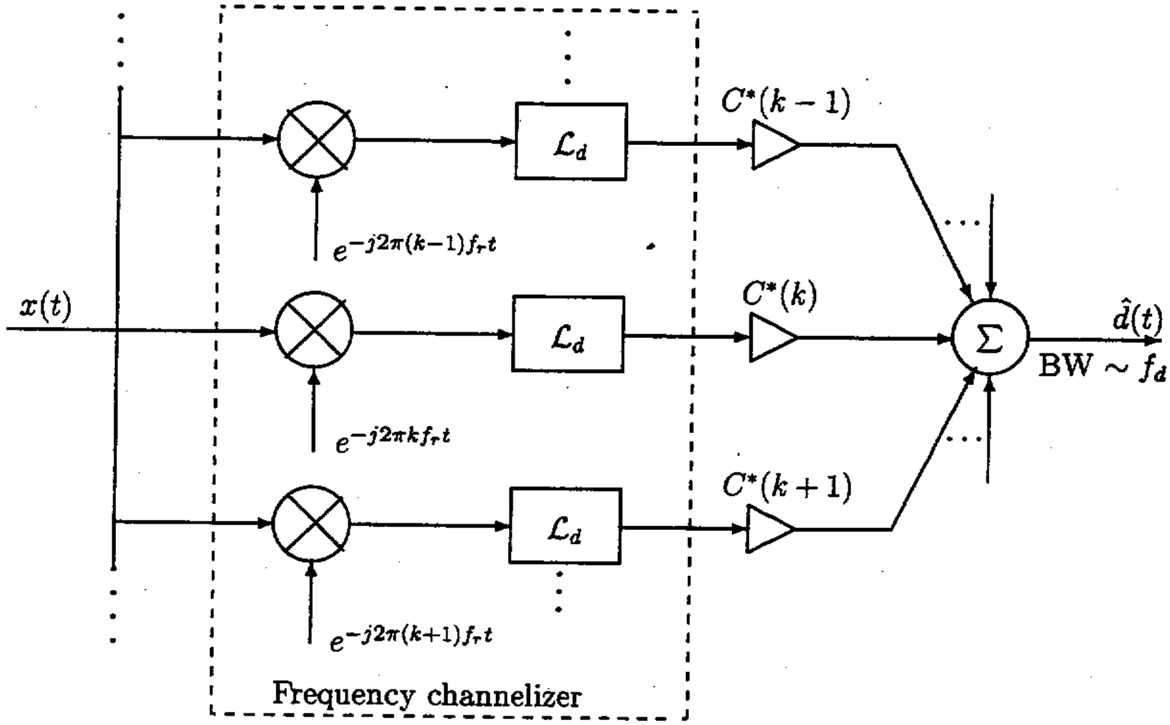

Figure 2: Fourier series representation of the DSSS despreader.

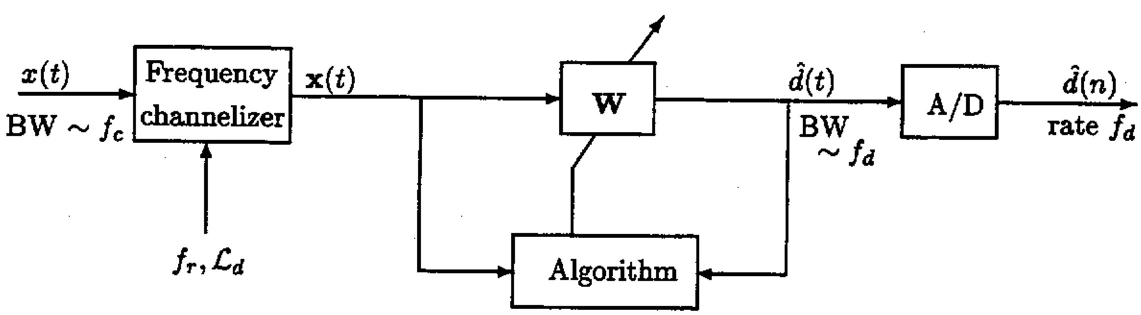

Figure 3: Blind despreader, frequency-domain representation.

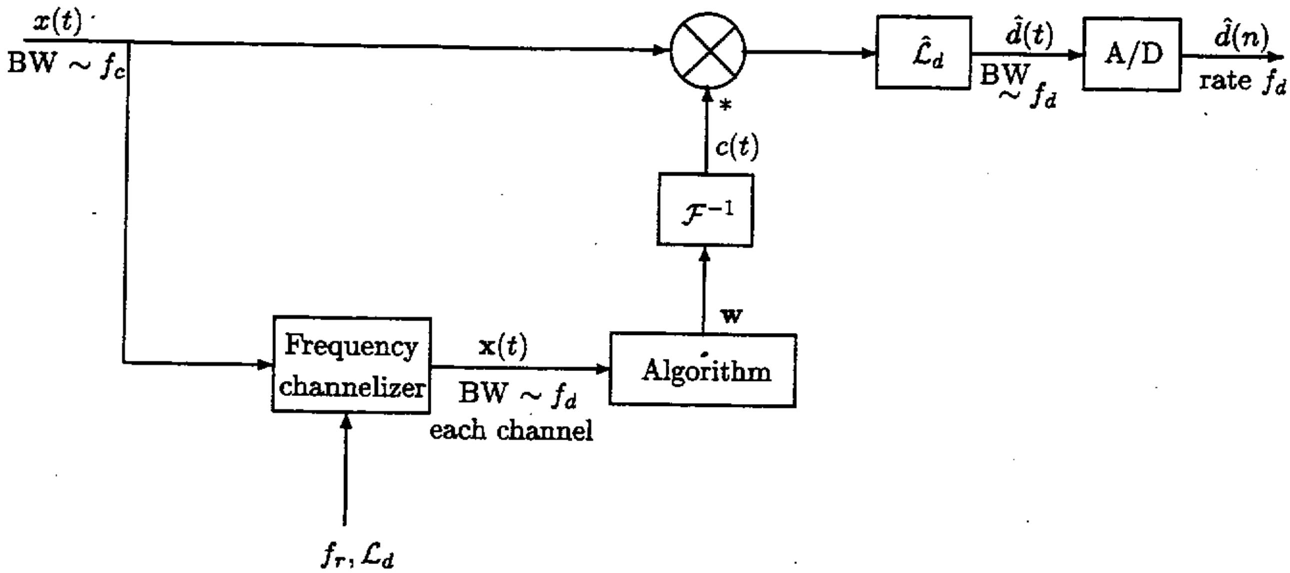

Figure 4: Blind despreader, time-domain representation.

If the filters are constrained by (9), then Figure 2 leads to the structure of Figure 3. Performing the frequency shift operation on the received signal results in the vector signal $\mathbf{x}(t) = [x_k(t)]_k$ which is linearly combined to obtain the despread signal

$$\begin{aligned} \hat{d}(t) &= \sum_k w^*(k) x_k(t) \\ &= \mathbf{w}^H \mathbf{x}(t), \end{aligned} \tag{10}$$

where $\mathbf{w} = [w(k)]$ and $(\cdot)^H$ denotes the conjugate-transpose (Hermitian transpose) operation. Figure 4 illustrates an implementation of the processor that resembles a conventional synchronizer. In this structure, the processor weights are used to estimate the spreading sequence, which is then used for despreading. This spreading sequence is synchronous with the spreading waveform component in the received signal and hence eliminates the synchronization process carried out in conventional receivers. The algorithm for the computation of $\mathbf{w}$ can be the same as in Figure 3. The despreading is carried out in the time-domain by

$$\hat{d}(t) = \mathcal{L}_d \circ [\hat{c}^*(t) x(t)], \tag{11}$$

where $\hat{c}(t)$ is obtained from $\mathbf{w}$ by the Fourier relationship

$$\hat{c}(t) = \sum_k w(k) e^{j2\pi k f_r t}. \tag{12}$$

2.2 Algorithm Development

The frequency-channelized DSSS signal $\mathbf{s}(t) = [s_k(t)]_k$ is given by

$$s_k(t) = \mathcal{L}_d \circ [s(t) e^{-j2\pi k f_r t}]$$

$$= \mathcal{L}_d \circ [\sum_l C(l)d(t)e^{-j2\pi(k-l)f_r t}], \tag{13}$$

using (1) and (3). Letting $k - l = m$, we have $l = k - m$ and (13) may be rewritten as

$$\begin{aligned} s_k(t) &= \sum_m C(k-m)\{\mathcal{L}_d \circ [d(t)e^{-j2\pi m f_r t}]\} \\ &= \sum_m C(k-m)d_m(t). \end{aligned} \tag{14}$$

Then

$$\begin{aligned} \mathbf{s}(t) &= [s_k(t)]_k \\ &= \sum_m \mathbf{c}_m d_m(t) \\ &= \mathbf{C}\mathbf{d}(t) \end{aligned} \tag{15}$$

where $\mathbf{d}(t) = [d_k(t)]_k$ is the frequency-channelized *message* signal and $\mathbf{C}$ is the matrix of Fourier coefficients of the spreading code. That is,

$$\begin{aligned} \mathbf{C} &= [C(k-m)]_{k,m} \\ &= [\ldots \mathbf{c}_{-1}\ \mathbf{c}_0\ \mathbf{c}_1 \ldots], \\ \mathbf{c}_m &= [C(k-m)]_k \\ &= \begin{bmatrix} \vdots \\ C(-1-m) \\ C(m) \\ C(1-m) \\ \vdots \end{bmatrix}. \end{aligned} \tag{16}$$

Similarly, the delayed, attenuated and Doppler-shifted DSSS signal $\tilde{s}(t) = ae^{j2\pi\Delta t}s(t-\tau)$ (where a is the attenuation, τ is the delay and Δ is the Doppler shift) has a frequency-channelized representation

$$\tilde{\mathbf{s}}(t) = \tilde{\mathbf{C}}\tilde{\mathbf{d}}(t) \tag{17}$$

$$\tilde{\mathbf{C}} = [\tilde{C}(k-m)]_{k,m} \tag{18}$$

$$\tilde{\mathbf{d}}(t) = [\mathcal{L}_d \circ (\tilde{d}(t)e^{-j2\pi k f_r t})]_k, \tag{19}$$

where

$$\begin{aligned} \tilde{d}(t) &= ae^{j2\pi\varepsilon_0 f_r t}d(t-\tau) \\ \tilde{C}(k) &= C(k-k_0)e^{-j2\pi(k-k_0)f_r\tau} \\ \tilde{c}(t) &= c(t-\tau)e^{j2\pi k_0 f_r t} \end{aligned}$$

and where $\Delta = (k_0 + \varepsilon_0)f_r$, and k_0 has an integer value such that $|\varepsilon_0| \leq 1/2$. The received data signal $x(t)$ therefore has the frequency-channelized representation

$$\mathbf{x}(t) = \tilde{\mathbf{C}}\tilde{\mathbf{d}}(t) + \mathbf{i}(t) \tag{20}$$

in the general *single signal environment* where the frequency-channelizer receives a single attenuated, delayed and Doppler-shifted DSSS signal in background noise $i(t)$, and $\mathbf{i}(t)$ is the frequency-channelized noise-component,

$$x(t) = ae^{j2\pi\Delta t}s(t-\tau) + i(t). \tag{21}$$

The autocorrelation matrix of $\mathbf{x}(t)$ is

$$\hat{\mathbf{R}}_{\mathbf{xx}} = \langle \mathbf{x}(t)\,\mathbf{x}^H(t)\rangle_T,$$

where $\langle\cdot\rangle_T$ denotes time-averaging over the reception interval $[0,T)$.

$$\begin{aligned}\hat{\mathbf{R}}_{\mathbf{xx}} &= \langle[\tilde{\mathbf{C}}\tilde{\mathbf{d}}(t)+\mathbf{i}(t)][\tilde{\mathbf{C}}\tilde{\mathbf{d}}(t)+\mathbf{i}(t)]^H\rangle_T \\ &= \tilde{\mathbf{C}}\langle\tilde{\mathbf{d}}(t)\tilde{\mathbf{d}}^H(t)\rangle_T\tilde{\mathbf{C}}^H + \tilde{\mathbf{C}}\langle\tilde{\mathbf{d}}(t)\mathbf{i}^H(t)\rangle_T \\ &\quad +\langle\mathbf{i}(t)\tilde{\mathbf{d}}^H(t)\rangle_T\tilde{\mathbf{C}}^H + \langle\mathbf{i}(t)\mathbf{i}^H(t)\rangle_T.\end{aligned} \tag{22}$$

If $d(t)$ and $i(t)$ are statistically independent, then as the averaging time grows to infinity, the autocorrelation converges to

$$\hat{\mathbf{R}}_{\mathbf{xx}} = \tilde{\mathbf{C}}\hat{\mathbf{R}}_{\tilde{\mathbf{d}}\tilde{\mathbf{d}}}\tilde{\mathbf{C}}^H + \mathbf{R}_{\mathbf{ii}}. \tag{23}$$

Using the theory of spectral correlation [9, 10], the identity

$$\begin{aligned} u_k(t) &= \mathcal{H}\circ[u(t)e^{j2\pi kf_rt}] \\ \Rightarrow R_{u_ku_l} &\triangleq \langle u_k(t)u_l^*(t)\rangle_T \\ &= \int_{-\infty}^{\infty} S_{u_ku_l}(f)\,df, \\ S_{u_ku_l}(f) &= |H(f)|^2 S_{uu}^{\alpha}(f_1)\end{aligned} \tag{24}$$

where $S_{uu}^{\alpha}(f)$ is the *cyclic spectrum* of $u(t)$, α being the set of *cyclic frequencies*:

$$\begin{aligned}\alpha &= (k-l)f_r \\ f_1 &= f + \frac{(k+l)}{2}f_r.\end{aligned}$$

Thus (24) becomes

$$R_{u_ku_l} = \int_{-\infty}^{\infty} |H(f)|^2 S_{uu}^{(k-l)f_r}\left(f+\frac{(k+l)}{2}f_r\right)\,df \tag{25}$$

Equation (25) holds for arbitrary finite power waveform $u(t)$ and LTI filter $\mathcal{H}$, where $S_{uu}^{\alpha}(f)$ has zero value at every nonzero value of α if $u(t)$ is stationary, and nonzero value at discrete nonzero values of α if $u(t)$ exhibits second-order cyclostationarity at those values of α.

If the background noise $i(t)$ is temporally stationary and white, then the cyclic spectrum of $i(t)$ is given by

$$S_{ii}^{\alpha}(f) = \begin{cases} N_0, & \alpha = 0 \\ 0, & \text{otherwise.}\end{cases} \tag{26}$$

Then it follows from (25) that

$$R_{i_k i_l} = \begin{cases} R_{nn}, & k=l \\ 0, & \text{otherwise} \end{cases}$$
$$= R_{nn}\delta_{k-l} \tag{27}$$

where

$$R_{nn} = N_0 \int_{-\infty}^{\infty} |L_d(f)|^2 \, df \tag{28}$$

for any LPF frequency response $L_d(f)$, and δ_k is the Kronecker delta function. Similarly, if $d(t)$ is stationary with power spectrum $S_{dd}(f)$, then the cyclic spectrum of $\tilde{d}(t)$ is given by

$$S^{\alpha}_{\tilde{d}\tilde{d}}(f) = \begin{cases} |a|^2 S_{dd}(f - \varepsilon_0 f_r), & \alpha = 0 \\ 0, & \text{otherwise,} \end{cases} \tag{29}$$

and (25) can be used to show that

$$R_{\tilde{d}_k \tilde{d}_l} = R_{\tilde{d}_k \tilde{d}_k}\delta_{k-l} \tag{30}$$
$$R_{\tilde{d}_k \tilde{d}_k} = \int_{-\infty}^{\infty} |L_d(f)|^2 S_{dd}(f - \varepsilon_0 f_r + k f_r)\, df \tag{31}$$

for any LPF frequency response $L_d(f)$. Then substituting (27) and (30) into (23) yields the autocorrelation matrix

$$\hat{\mathbf{R}}_{\mathbf{xx}} = \tilde{\mathbf{C}} \operatorname{diag}\{R_{\tilde{d}_k \tilde{d}_k}\} \tilde{\mathbf{C}}^H + R_{nn}\mathbf{I}. \tag{32}$$

Equations (30)–(31) also hold if $d(t)$ is nonstationary but does not possess second-order cyclostationarity at any multiple of f_r. In particular, if $d(t)$ is a PSK signal with data rate f_d, power R_d, pulse shaping $P_d(f)$ and temporally white message sequence $d(n)$, then the cyclic spectrum of $\tilde{d}(t)$ is given by [10]

$$S^{\alpha}_{\tilde{d}\tilde{d}}(f) = |a|^2 S^{\alpha}_{dd}(f + \varepsilon_0 f_r) e^{-j2\pi\alpha\tau}$$
$$S^{\alpha}_{dd}(f) = \begin{cases} R_d P_d(f + \frac{k}{2} f_d) P^*_d(f - \frac{k}{2} f_d), & \alpha = k f_d \\ 0, & \text{otherwise,} \end{cases} \tag{33}$$

and (25) can be used to show that $R_{\tilde{d}_k \tilde{d}_l}$ is given by (30)–(31) if the message and code-repeat rates are *incommensurate*, such that f_d/f_r is irrational. This condition can also be shown to approximately hold if f_d and f_r are *practically incommensurate* such that $f_d/f_r = p/q$ where p and q are large relatively-prime integers. In this case, $R_{\tilde{d}_k \tilde{d}_l}$ reduces to

$$R_{\tilde{d}_k \tilde{d}_l} = \begin{cases} |a|^2 e^{j2\pi i f_r \tau} R_d \int_{-\infty}^{\infty} |L_d(f - \varepsilon_0 f_r - k f_r)|^2 P_d(f) P^*(f - ipf_r)\, df, & l = k + ip \\ 0, & \text{otherwise} \end{cases} \tag{34}$$
$$\approx \left[|a|^2 R_d \int_{-\infty}^{\infty} |L_d(f - \varepsilon_0 f_r - k f_r)|^2 |P_d(f)|^2 \, df \right] \delta_{k-l} \tag{35}$$

if qf_d (pf_r) is larger than the effective bandwidth of $P_d(f)$, and (32) holds for this signal as well.

If $c(t)$ has a constant modulus, then Parseval's relation can be used to further simplify (32) by noting that the Fourier coefficients of the spreading code satisfy the *shift-orthonormality property*:

$$\begin{aligned}\sum_k C(k)C^*(k-l) &= \sum_k \frac{1}{T_r}\int_{-T_r/2}^{T_r/2} c(t)e^{-j2\pi k f_r t}\,dt\, C^*(k-l) \\ &= \frac{1}{T_r}\int_{-T_r/2}^{T_r/2} e^{-j2\pi l f_r t}\,dt \\ &= \delta_l \end{aligned} \tag{36}$$

Applying (36) to the Fourier coefficient matrix $\mathbf{C}$ yields

$$\mathbf{C}^H\mathbf{C} = \mathbf{I} \tag{37}$$

Equations (36)–(37) hold for *any* spreading waveform as long as it has a constant modulus over the entire code period. Thus (37) also applies to the frequency-shifted Fourier coefficient matrix $\tilde{\mathbf{C}}$.

Then it follows from (32) that the Fourier coefficients of $\tilde{c}(t)$ can be computed from the eigenstructure of $\hat{\mathbf{R}}_{\mathbf{xx}}$. Since $\tilde{\mathbf{C}}$ is a unitary matrix satisfying the orthonormality property (37), according to the theory of matrices, the eigenvectors of the autocorrelation matrix are equal to $\tilde{\mathbf{C}}$ multiplied by the eigenvectors of $\text{diag}\{R_{\tilde{d}_k\tilde{d}_k}\}$, which are equal to the set of unit vectors $\{\mathbf{e_k}\}$. Thus the eigenvectors of $\hat{\mathbf{R}}_{\mathbf{xx}}$ are equal to $\{\tilde{\mathbf{C}}\mathbf{e}_k\} = \{\tilde{\mathbf{c}}_k\}$. Hence,

$$\begin{aligned}\hat{\mathbf{R}}_{\mathbf{xx}}\tilde{\mathbf{c}}_l &= (R_{nn}\mathbf{I} + \tilde{\mathbf{C}}\text{diag}\{R_{\tilde{d}_k\tilde{d}_k}\}\tilde{\mathbf{C}}^H)\tilde{\mathbf{c}}_l & (38)\\ &= R_{nn}\tilde{\mathbf{c}}_l + \tilde{\mathbf{C}}\text{diag}\{R_{\tilde{d}_k\tilde{d}_k}\}\tilde{\mathbf{C}}^H\tilde{\mathbf{c}}_l & \\ &= (R_{nn} + R_{\tilde{d}_l\tilde{d}_l})\tilde{\mathbf{c}}_l & (39)\\ \Rightarrow \lambda_l &= R_{nn} + R_{\tilde{d}_l\tilde{d}_l} & (40)\end{aligned}$$

Furthermore, if $\tilde{d}(t)$ is a PSK waveform and $L_d(f)$ is set equal to $P_d(f+\varepsilon_0 f_r)$ (or $L_d(f)$ is set equal to $P_d(f)$ and $\varepsilon_0 f_r \ll f_d$), then (38) is maximized for $l = 0$ and the k_0-shifted Fourier expansion of the code in the received signal can be estimated from the *dominant mode* (eigenvector associated with the maximum eigenvalue) of $\hat{\mathbf{R}}_{\mathbf{xx}}$. The dominant mode can also be used to directly estimate $\tilde{d}(t)$ from $\mathbf{x}(t)$ using (10), or it can be used to estimate $\tilde{c}(t)$, and despread the data signal using (2). For this reason, this technique is referred to here as the *dominant mode despreading* (DMDS) algorithm.

The basic DMDS algorithm may be summarized as follows.

1. Frequency-channelize the received data signal $x(t)$ to form the vector signal $\mathbf{x}(t)$, using the formula

   $$\mathbf{x}(t) = [\mathcal{L}_d \circ (x(t)e^{-j2\pi k f_r t})]_k,$$

   where f_r is the code-repetition rate and $\mathcal{L}_d$ is an LTI lowpass filter that is matched to the pulse shape of the message signal.

2. Compute the autocorrelation matrix of $\mathbf{x}(t)$ using the formula

$$\hat{\mathbf{R}}_{\mathbf{xx}} = \langle \mathbf{x}(t)\mathbf{x}^H(t)\rangle_T$$

where $\langle\cdot\rangle_T$ denotes the time-averaging operation over the collection interval $[0, t)$, and $(\cdot)^H$ denotes the conjugate-transpose operation.

3. Compute the *dominant mode* of $\hat{\mathbf{R}}_{\mathbf{xx}}$ by solving for the maximum eigenvalue λ_{max} and the associated eigenvector $\mathbf{w}_{max}$ of the eigenequation

$$\hat{\mathbf{R}}_{\mathbf{xx}}\mathbf{w} = \lambda\mathbf{w}.$$

4. Compute the processor output signal using either of the formulas

$$\hat{d}(t) = \mathbf{w}_{max}^H\mathbf{x}(t) \qquad \text{frequency domain despreader,}$$
$$\left.\begin{array}{l} \hat{d}(t) = \hat{\mathcal{L}}_d \circ [\hat{c}^*(t)x(t)] \\ \hat{c}(t) = \sum_k w_{max}(k)e^{j2\pi kf_rt} \end{array}\right\} \text{time-domain despreader,}$$

where $\hat{\mathcal{L}}_d$ is an LTI lowpass filter (not necessarily equal to $\mathcal{L}_d$) that is matched in some way to the pulse shape on the message signal.

The technique provides a delayed and Doppler-shifted version of the message signal, since the algorithm estimates $\tilde{c}(t)$ (or $\tilde{C}(k)$) rather than $c(t)$. This requires keeping track of the timing of the collected data in order to correctly use the estimated code. In addition, the DMDS algorithm automatically down-converts $\hat{d}(t)$ to within $\pm f_r/2$ carrier cycles by subsuming the component of the carrier offset with periodicity f_r into the spreading code. Thus, the estimation of $\tilde{c}(t)$ is insensitive to carrier offsets beyond $\pm f_r/2$ carrier cycles and it may still be necessary to perform a fine tuning carrier-recovery procedure to fully recover the message signal, however, this can be carried out at the *despread* SNR.

3 Application of the DMDS Algorithm in a Multipath Environment

Multipath propagation is inevitable in the wireless environment, so it is important to see how the algorithm performs in this environment. A two path multipath model is used here:

$$x(t) = s(t) + g\,s(t-\tau) + i(t), \tag{41}$$

where g is the complex scaling factor of the multipath component and τ is the delay. Then the frequency-channelized signal is given by

$$\begin{aligned} x_k(t) &= \mathcal{L}_d \circ [x(t)e^{-j2\pi kf_rt}] \\ &= \mathcal{L}_d \circ [(c(t)d(t) + g\,c(t-\tau)d(t-\tau) + i(t))e^{-j2\pi kf_rt}] \\ &= \sum_m C(k-m)d_k(t) + \sum_m \tilde{C}(k-m)\tilde{d}_k(t) + i_k(t), \end{aligned} \tag{42}$$

where

$$\begin{aligned}\tilde{C}(k) &= C(k)e^{-j2\pi k f_r \tau} \\ \tilde{d}_k(t) &= \mathcal{L}_d \circ [g\, d(t-\tau)e^{-j2\pi k f_r t}].\end{aligned}$$

Then,

$$\begin{aligned}\mathbf{x}(t) &= [x_k(t)]_k \\ &= \mathbf{C}\mathbf{d}(t) + \tilde{\mathbf{C}}\tilde{\mathbf{d}}(t) + \mathbf{i}(t) \\ \tilde{\mathbf{C}} &= [\tilde{C}(k-m)]_{k,m} \\ \tilde{\mathbf{d}}(t) &= [\tilde{d}_k(t)]_k.\end{aligned} \tag{43}$$

Then the autocorrelation matrix of $\mathbf{x}(t)$ is

$$\begin{aligned}\hat{\mathbf{R}}_{\mathbf{xx}} &= \langle \mathbf{x}(t)\mathbf{x}^H(t)\rangle_T \\ &= \langle \mathbf{C}\mathbf{d}(t)\mathbf{d}^H(t)\mathbf{C}^H + \mathbf{C}\mathbf{d}(t)\tilde{\mathbf{d}}^H(t)\tilde{\mathbf{C}}^H \\ &\quad + \tilde{\mathbf{C}}\tilde{\mathbf{d}}(t)\mathbf{d}^H(t)\mathbf{C}^H + \tilde{\mathbf{C}}\tilde{\mathbf{d}}(t)\tilde{\mathbf{d}}^H(t)\tilde{\mathbf{C}}^H + \mathbf{i}(t)\mathbf{i}^H(t)\rangle_T\end{aligned} \tag{44}$$

assuming that $d(t)$ and $i(t)$ are temporally independent.

$$\hat{\mathbf{R}}_{\mathbf{xx}} = \mathbf{C}\mathbf{R}_{\mathbf{dd}}\mathbf{C}^H + \mathbf{C}\mathbf{R}_{\mathbf{d}\tilde{\mathbf{d}}}\tilde{\mathbf{C}}^H + \tilde{\mathbf{C}}\mathbf{R}_{\tilde{\mathbf{d}}\mathbf{d}}\mathbf{C}^H + \tilde{\mathbf{C}}\mathbf{R}_{\tilde{\mathbf{d}}\tilde{\mathbf{d}}}\tilde{\mathbf{C}}^H + \mathbf{R}_{\mathbf{ii}} \tag{45}$$

as the averaging time grows to infinity.

$$R_{d_k\tilde{d}_l} = \int_{-\infty}^{\infty} S_{d_k\tilde{d}_l}(f)\,df, \tag{46}$$

$$S_{d_k\tilde{d}_l}(f) = |L_d(f)|^2 S_{u_k v_l}(f), \tag{47}$$

$$u_k(t) = d(t)e^{-j2\pi k f_r t} \tag{48}$$

$$v_l(t) = g\, d(t-\tau)e^{-j2\pi l f_r t} \tag{49}$$

$$S_{u_k v_l}(f) = \mathcal{F}\{\langle u_k\left(t+\frac{\nu}{2}\right) v_l^*\left(t-\frac{\nu}{2}\right)\rangle_T\}_\nu$$

where $\mathcal{F}\{\cdot\}_\nu$ is used to denote Fourier transformation with ν as the variable of integration.

$$\begin{aligned}S_{u_k v_l}(f) &= \mathcal{F}\{\langle d\left(t+\frac{\nu}{2}\right)e^{-j2\pi k f_r\left(t+\frac{\nu}{2}\right)} \\ &\quad \times \left[g\, d\left(t-\tau-\frac{\nu}{2}\right)e^{-j2\pi l f_r\left(t-\tau-\frac{\nu}{2}\right)}\right]^* e^{j2\pi l f_r \tau}\rangle_T\}_\nu.\end{aligned}$$

Letting $t-\frac{\tau}{2}=t_1$ and $\frac{\tau}{2}+\frac{\nu}{2}=\frac{\lambda}{2}$, it can be shown [11] that

$$\begin{aligned}S_{u_k v_l}(f) &= g^*\mathcal{F}\{\langle d\left(t_1+\frac{\lambda}{2}\right)e^{-j2\pi k f_r\left(t_1+\frac{\lambda}{2}\right)} \\ &\quad \times \left[d\left(t_1-\frac{\lambda}{2}\right)e^{-j2\pi l f_r\left(t_1-\frac{\lambda}{2}\right)}\right]^*\rangle_T\}_\nu e^{j2\pi l f_r \tau} \\ &= g^* S_{dd}^{\alpha}(f_1)e^{j2\pi l f_r \tau},\end{aligned} \tag{50}$$

where

$$\alpha = (k-l)f_r$$
$$f_1 = f + \frac{(k+l)}{2} f_r.$$

Thus (46) becomes

$$\begin{aligned} R_{d_k \tilde{d}_l} &= g^* e^{j2\pi l f_r \tau} \int_{-\infty}^{\infty} |L_d(f)|^2 S_{dd}^{(k-l)f_r} \left(f + \frac{(k+l)}{2} f_r \right) df \\ &= g^* e^{j2\pi l f_r \tau} R_{d_k d_l} \end{aligned} \quad (51)$$

where use has been made of (25). Since

$$S_{dd}^{\alpha}(f) = \begin{cases} S_{dd}(f), & \alpha = 0 \\ 0, & \text{otherwise,} \end{cases} \quad (52)$$

$$\begin{aligned} R_{d_k \tilde{d}_l} &= R_{d_k \tilde{d}_k} \delta_{k-l} \\ &= g^* e^{j2\pi l f_r \tau} R_{d_k d_k} \delta_{k-l}. \end{aligned} \quad (53)$$

Then

$$\begin{aligned} \mathbf{R}_{\mathbf{d}\tilde{\mathbf{d}}} &= \text{diag}\{R_{d_k \tilde{d}_k}\} \\ &= g^* \text{diag}\{e^{j2\pi k f_r \tau} R_{d_k d_k}\} \\ &= g^* \mathbf{R}_{\mathbf{dd}} \mathbf{E}^H, \end{aligned} \quad (54)$$

where

$$\mathbf{E} \triangleq \text{diag}\{e^{-j2\pi k f_r \tau}\} \quad (55)$$

In a similar fashion, it can be shown that

$$\mathbf{R}_{\tilde{\mathbf{d}}\mathbf{d}} = g\, \mathbf{E}\, \mathbf{R}_{\mathbf{dd}} \quad (56)$$

and

$$\mathbf{R}_{\tilde{\mathbf{d}}\tilde{\mathbf{d}}} = |g|^2\, \mathbf{E}\, \mathbf{R}_{\mathbf{dd}}\, \mathbf{E}^H. \quad (57)$$

Using (54), (56) and (57) in (45),

$$\begin{aligned} \hat{\mathbf{R}}_{\mathbf{xx}} &= \mathbf{C}\mathbf{R}_{\mathbf{dd}}\mathbf{C}^H + g^* \mathbf{C}\mathbf{R}_{\mathbf{dd}}\, \mathbf{E}^H \tilde{\mathbf{C}}^H + g\, \tilde{\mathbf{C}}\mathbf{E}\, \mathbf{R}_{\mathbf{dd}}\mathbf{C}^H + |g|^2\, \tilde{\mathbf{C}}\mathbf{E}\, \mathbf{R}_{\mathbf{dd}}\, \mathbf{E}^H \tilde{\mathbf{C}}^H + \mathbf{R}_{\mathbf{ii}} \\ &= (\mathbf{C} + g\, \tilde{\mathbf{C}}\mathbf{E})\mathbf{R}_{\mathbf{dd}}(\mathbf{C} + g\, \tilde{\mathbf{C}}\mathbf{E})^H + \mathbf{R}_{\mathbf{ii}}. \end{aligned} \quad (58)$$

The columns of $(\mathbf{C} + g\tilde{\mathbf{C}}\mathbf{E})$ are $\{\mathbf{c}_m + g\tilde{\mathbf{c}}_m\, e^{-j2\pi m f_r \tau}\}$,

$$\begin{aligned} \mathbf{c}_m &= [C(k-m)]_k, \\ \tilde{\mathbf{c}}_m &= [\tilde{C}(k-m)]_k. \end{aligned}$$

In this case, the shift-orthonormality property is only approximately satisfied. The dominant mode corresponds to $m = 0$. The eigenvector of $\hat{\mathbf{R}}_{\mathbf{xx}}$ associated with the maximum eigenvalue is approximately equal to $\mathbf{c}_0 + g\,\tilde{\mathbf{c}}_0$ and the (approximate) estimated code is obtained from its inverse Fourier transform,

$$\begin{aligned}\hat{c}(t) &\approx \mathcal{F}^{-1}\{\mathbf{c}_0 + g\,\tilde{\mathbf{c}}_0\} \\ &= c(t) + g\,c(t-\tau). \end{aligned} \tag{59}$$

The demodulated signal is given by (4),

$$\begin{aligned}\hat{d}(t) &\approx \hat{\mathcal{L}}_d \circ [(c^*(t) + g\,c^*(t-\tau))(c(t)d(t) + g\,c(t-\tau)d(t-\tau) + i(t))] \\ &= \hat{\mathcal{L}}_d \circ [c^*(t)c(t)d(t)] + \hat{\mathcal{L}}_d \circ [g\,c^*(t)c(t-\tau)d(t-\tau)] + \hat{\mathcal{L}}_d \circ [c^*(t)i(t)] \\ &\quad + \hat{\mathcal{L}}_d \circ [g^*\,c^*(t-\tau)c(t)d(t)] + \hat{\mathcal{L}}_d \circ [|g|^2\,c^*(t-\tau)c^*(t-\tau)d(t-\tau)] \\ &\quad + \hat{\mathcal{L}}_d \circ [g^*\,c^*(t-\tau)i(t)]. \end{aligned} \tag{60}$$

If the spreading code has good autocorrelation properties [1], then for $\tau > T_c$, the second and third terms are negligible. Assuming that the spreading code and noise are statistically independent, the third and the sixth terms approach zero. Furthermore, $|c(t)|^2 = |c(t-\tau)|^2 = 1$. Hence (60) becomes

$$\hat{d}(t) \approx d(t) + |g|^2\,d(t-\tau). \tag{61}$$

Thus, the estimated data also contains a delayed and attenuated component. In fact, the attenuation of the delayed component is greater than that in the received signal as indicated by the factor $|g|^2$. When this factor is not negligible, equalization of the data can be carried out. A point worth noting is that equalization needs to be performed only at the despread SNR and the data rate since the eigenstructure technique automatically synchronizes with the multipath component also.

4 Conclusions

The eigenstructure technique for synchronization of DSSS signals is presented. The algorithm is developed and it is shown that under infinite time-average assumptions, a perfect estimate of the spreading code can be obtained for arbitrary levels of white background noise. The only requirements are that the code have a constant modulus and that the data and code repeat rates be practically incommensurate. The insensitivity of the technique to arbitrary frequency and time offsets is also proven.

The technique is analyzed when applied to a signal received in a static multipath environment. It is shown the the technique provides a multipath estimate of the spreading code, which can be exploited by employing an equalizer on the despread data at the despread SNR and at the data rate. Extensive simulation results are presented in [5, 6].

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